Precalculus

Maryland College and Career Ready Standards for Mathematics

Overview

The Maryland College and Career Ready (MCCRS) Precalculus standards define what students should understand and be able to do in their study of Precalculus. The standards are divided into two sections, Precalculus A and Precalculus B. The standards included in Precalculus A represent the skills and understandings a student must possess to be successful in Calculus AB, Business Calculus, or most Calculus I courses offered by colleges. The standards included in Precalculus B represent additional skills and understandings a student must possess to be successful in Calculus BC and in other college-level mathematics. For a more in-depth look at the College Board's requirements for Advanced Placement Calculus AB and BC refer to the document accessed via the link College Board Exam Descriptions.

Precalculus A

In Precalculus A, students synthesize their conceptual understanding of algebraic and transcendental function families. Understanding of functions is applied to solving real world problems that require students to build and/or interpret functions. Students also improve their understandings of the properties of mathematics that allow them to hone their ability to manipulate algebraic expressions, equations and inequalities.

Precalculus B

The skills and understandings developed in Precalculus B provide the prerequisites needed for Calculus BC and other college level mathematics coursework such as analytic geometry, linear algebra, discrete mathematics and vector calculus. Coursework includes additional topics related to complex numbers, matrices, vectors, conic sections, parametric equations, polar coordinates and equations, sequences and series, and an introduction to limits*.

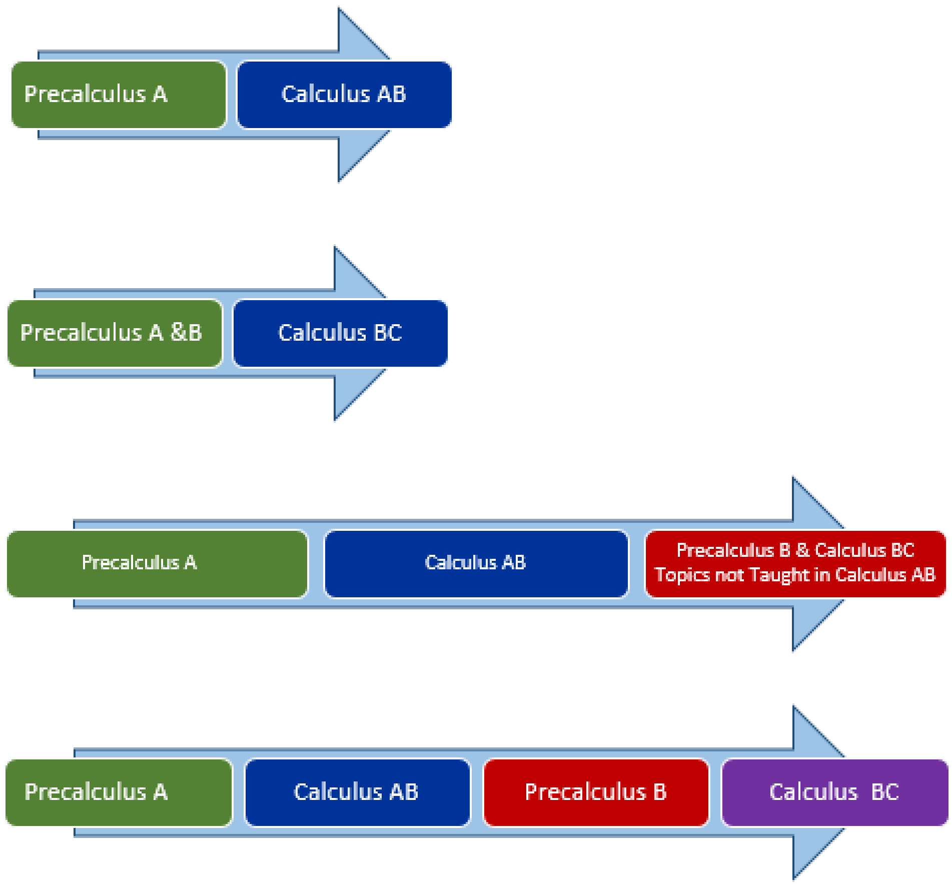
***Note:**

Depending on the structure a district selects, placement of **Logistics Growth** and Introduction of Limits could be placed in Precalculus A or Precalculus B. Districts should consider pacing, timing, and the sequence of courses (see next section) to determine the placement of these topics.

Course Sequences

As with all Maryland standards, the Precalculus standards do not represent a curriculum and do not suggest pedagogy. The Precalculus standards are intended to serve as guidance to Maryland districts as they develop Precalculus curricular documents.

Examples of Possible Course Sequences:



Understanding the Precalculus Standards Document

The standards tables include:

- Standards with Clarifications/Examples and Function Families

  Many standards apply to multiple function families and are therefore taught multiple times in Algebra I, Algebra II and Precalculus. Below each standard is a list of the various function families to which each standard applies. Clarifications provide insight as to how to extend prior learning associated with standards. Clarifications and examples are not provided for standards where added clarity is not needed.

- Plus standards

  The MCCRS for Mathematics include standards referred to as "plus standards". These are labeled with a plus sign (+). They are not included in Algebra I, Geometry or Algebra II as these standards were represent mathematics needed for advanced level mathematics coursework. The plus standards have been included in either the Precalculus A or Precalculus B standards. The plus sign has been removed from the name of the standard.

- Modeling standards

  Modeling standards are designated by the use of a star symbol (★) after the name of the standard. Modeling is a mathematical process of choosing and applying appropriate mathematics to analyze, understand, and solve real world problems. Modeling standards are not a set of isolated topics within their own conceptual category, but rather a collection of standards that connect to the modeling cycle.

- New standards

  Standards not originally included in the MCCRS for Mathematics but were added are designated by an underlined code.

- Standards for Mathematical Practice

  1. Make sense of problems and persevere in solving them.
  2. Reason abstractly and quantitatively.
  3. Construct viable arguments and critique the reasoning of others.
  4. Model with mathematics.
  5. Use appropriate tools strategically.
  6. Attend to precision.
  7. Look for and make use of structure.
  8. Look for and express regularity in repeated reasoning.

Precalculus A

Conceptual Category: Number and Quantity

DOMAIN: THE COMPLEX NUMBER SYSTEM (N.CN)

Cluster: C. Use complex numbers in polynomial identities and equations.

N.CN.C.8 Extend polynomial identities to the complex numbers. For example, rewrite x^2+4 as $(x+2i)(x-2i)$.

- **Clarifications/Examples:**
  - Apply the Complex Conjugate Theorem when:
    - solving polynomial equations with degrees greater than or equal to two.
    - analyzing the graph of a polynomial.
    - building a polynomial given one complex root.
- **Function families to which this standard applies:**
  - Polynomial Functions
  - Rational Functions

N.CN.C.9 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

- **Clarifications/Examples:**
  - Apply knowledge of Fundamental Theorem of Algebra to solving polynomial equations of degree 3 and higher.
  - Students should be able to identify existence of complex roots.
  - Make connections between the nature of the roots of an equation and the behavior of the graph of the function.
- **Function families to which this standard applies:**
  - Polynomial Functions
  - Rational Functions

Conceptual Category: Algebra

DOMAIN: SEEING STRUCTURE IN EXPRESSIONS (A.SSE)

Cluster: A. Interpret the structure of expressions.

A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it.

- **Clarifications/Examples:**
  - Use factors of polynomials to simplify/analyze rational expressions and the graphs of the related functions.
  - Factor polynomial expressions, of degree three and higher, completely, over the complex number system (e.g. Factor $x^4 - 3x^2 - 28$ to $(x^2 + 4)(x^2 - 7)$ and then to $(x + 2i)(x - 2i)(x + \sqrt{7})(x - \sqrt{7})$.
  - Rewrite trigonometric expressions based on algebraic structures (e.g. $\sin^3 x + 1$; $\frac{\sin^4 x - \cos^4 x}{\sin^2 x - \cos^2 x}$).
  - Discuss conjugates and how their structure can help when simplifying expressions, verifying identities and solving equations.
  - Factor expressions to include using the sum and difference of cubes (e.g. Factor $x^6 - 27y^3$).
  - Recognize and factor an expression in quadratic form (e.g. $e^{2x} - 9e^x + 14$, $2\cos^2 x - 3\cos x + 1$, $x^6 - 9x^3 + 8$).
- **Function families to which this standard applies:**
  - Logarithmic/Exponential Functions
  - Polynomial Functions
  - Radical Power Functions
  - Rational Functions
  - Trigonometric Functions

Cluster: B. Write expressions in equivalent forms to solve problems.

A.SSE.B.3★ Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

- **Clarifications/Examples:**
  - Produce an equivalent form of a rational expression to reveal information about the behavior of the graph of the related function.
    - number and type of discontinuities
    - zeros
    - asymptotes and holes
  - Rewrite complex fractions as rational expressions $\left(e.g. Rewrite \frac{\frac{1}{x} - \frac{1}{x^2}}{\frac{2}{x} + \frac{1}{x^2}} \ as \ \frac{x-1}{2x+1}\right)$.

- **Function families to which this standard applies:**
  - Logarithmic/Exponential Functions
  - Polynomial Functions
  - Radical Power Functions
  - Rational Functions
  - Trigonometric Functions

A.SSE.B.3.c★ Use the properties of exponents to transform expressions for exponential functions. For example, the expression $\mathbf{1.15^t}$ can be rewritten as $\left(\mathbf{1.15^{\frac{1}{12}}}\right)^{\mathbf{12t}} \approx \mathbf{1.012^{12t}}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

- **Clarifications/Examples:**
  - Model in context.
- **Function families to which this standard applies:**
  - Logarithmic/Exponential Functions

DOMAIN: ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSIONS (A.APR)

Cluster: B. Understand the relationship between zeroes and factors of polynomials.

A.APR.B.2 Know and apply the Remainder Theorem: For a polynomial $\boldsymbol{p(x)}$ and a number a, the remainder on division by $\boldsymbol{x - a}$ is $\boldsymbol{p(a)}$, so $\boldsymbol{p(a) = 0}$ if and only if $\boldsymbol{(x - a)}$ is a factor of $\boldsymbol{p(x)}$.

- **Clarifications/Examples:**
  - Factor polynomials to simplify rational expressions.
  - Use the Remainder Theorem to determine zeros (roots) of a polynomial.
- **Function families to which this standard applies:**
  - Polynomial Functions
  - Rational Functions

A.APR.B.3.a Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

- **Clarifications/Examples:**
  - Identify real zeros of polynomials.
  - Understand the relationship between the degree of a polynomial and the number and nature of the zeros of the polynomial.
- **Function families to which this standard applies:**
  - Polynomial Functions

Cluster: C. Use polynomial identities to solve problems.

A.APR.C.5 Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle.

- **Clarifications/Examples:**
    - Limit to $n \leq 5$, and binomials with variables coefficient of one, or constants less than four.
- **Function families to which this standard applies:**
    - Polynomial Functions

Cluster: D. Rewrite rational expressions.

A.APR.D.6 Rewrite simple rational expressions in different forms.

- **Clarifications/Examples:**
    - Rational expressions have no restrictions on degree of the numerator or denominator.
    - Understand the connection between rewriting rational expressions and using long division when factoring polynomials.
        - Apply the Rational Root Theorem.
- **Function families to which this standard applies:**
    - Polynomial Functions
    - Rational Functions

A.APR.D.7 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication and division by a nonzero rational expression; add, subtract, multiply and divide rational expressions.

- **Clarifications/Examples:**
    - Emphasize operations on rational expressions.
- **Function families to which this standard applies:**
    - Rational Functions

DOMAIN: CREATING EQUATIONS (A.CED)

Cluster: A. Create equations that describe numbers or relationships.

A.CED.A.1.a★ Create equations and inequalities in one variable and use them to solve problems.

- **Clarifications/Examples:**
    - Create equations and inequalities in one variable involving all algebraic and transcendental functions and piecewise defined functions that combine different types of functions.
    - Use equations and inequalities that arise from any type of function to solve problems.
- **Function families to which this standard applies:**
    - Logarithmic/Exponential Functions
    - Polynomial Functions
    - Radical Power Functions
    - Rational Functions
    - Trigonometric Functions

A.CED.A.1b★ Create polynomial equations given roots.

- **Clarifications/Examples:**
    - Use the Factor Theorem.
    - Use the Conjugate Root Theorem as it applies to irrationals.
- **Function families to which this standard applies:**
    - Polynomial Functions

A.CED.A.3★ Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

- **Clarifications/Examples:**
    - Understand why constraints exist for certain equations and inequalities and for systems of equations and inequalities.
    - Represent constraints of equations that contain composite expressions $\left(\text{e.g. } \sqrt{x^2-4} > 0\right)$.
- **Function families to which this standard applies:**
    - Logarithmic/Exponential Functions
    - Polynomial Functions
    - Radical Power Functions
    - Rational Functions
    - Trigonometric Functions

DOMAIN: REASONING WITH EQUATIONS AND INEQUALITIES (A.REI)

Cluster: B. Solve equations and inequalities on one variable.

A.REI.B Solve equations and inequalities in one variable.

- **Clarifications/Examples:**
  - Include equations and inequalities that contain combinations of various algebraic and transcendental functions.
  - Include equations that contain composite expressions.
  - Solve equations in one variable algebraically, numerically and/or graphically.
  - Justify solution methods.
- **Function families to which this standard applies:**
  - Logarithmic/Exponential Functions
  - Polynomial Functions
  - Radical Power Functions
  - Rational Functions
  - Trigonometric Functions

Cluster: C. Solve systems of equations.

A.REI.C.7a Solve systems of equations comprised of various combinations of all algebraic and transcendental functions in two variables.

- **Clarifications/Examples:**
  - Explore algebraic, numeric and graphical methods for solving systems of equations.
  - Explore systems comprised of functions from two different function families (e.g. Solve the system $y = x^2$ and $y = 2\cos x$).
- **Function families to which this standard applies:**
  - Logarithmic/Exponential Functions
  - Polynomial Functions
  - Radical Power Functions
  - Rational Functions
  - Trigonometric Functions

Conceptual Category: Functions

DOMAIN: INTERPRETING FUNCTIONS (F.IF)

Cluster: A. Understand the concept of a function and use function notation.

F.IF.A.2a Extend evaluating functions to include operations with composite functions, e.g. $f(x+2) - f(x)$.

- **Clarifications/Examples:**
  - Build prerequisite skills needed to simplify difference quotient (e.g. Given $f(x) = x^2 + 3x + 1$ find $f(x+h)$).

- **Function families to which this standard applies:**
    - Logarithmic/Exponential Functions
    - Polynomial Functions
    - Radical Power Functions
    - Rational Functions
    - Trigonometric Functions

Cluster: B. Interpret functions that arise in application in terms of context.

F.IF.B.4★ For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities; sketch graphs showing key features given a verbal description of the relationship.

- **Clarifications/Examples:**
    - Interpret the key features of the graph of any function in terms of context.
    - Explore rate of change over various size intervals.
    - Estimate points of inflection.
    - Describe the concavity on intervals.
    - Explain how to recognize asymptotic behavior given various representations of a function (algebraic, numeric and graphic).
- **Function families to which this standard applies:**
    - Logarithmic/Exponential Functions
    - Polynomial Functions
    - Radical Power Functions
    - Rational Functions
    - Trigonometric Functions

F.IF.B.5★ Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

- **Clarifications/Examples:**
    - Discuss the domain of all function types including composite and inverse functions.
- **Function families to which this standard applies:**
    - Logarithmic/Exponential Functions
    - Polynomial Functions
    - Radical Power Functions
    - Rational Functions
    - Trigonometric Functions

Cluster: C. Analyze functions using different representations.

F.IF.C.7f★ Graph all functions including piecewise-defined functions, step functions and absolute value functions

- **Clarifications/Examples:**
    - Sketch polynomials of higher degree from factored form, using *x*- and *y*-intercepts, end behavior and degree.
    - Describe end behavior using appropriate notation, i.e. given an equation, as $x \to \pm\infty$, $f(x) \to \pm\infty$.
    - Use information about end behavior of a polynomial to sketch and identify the possible degree of a polynomial.
    - Graph inverse trigonometric functions and identify the related principal values.
- **Function families to which this standard applies:**
    - Logarithmic/Exponential Functions
    - Polynomial Functions
    - Radical Power Functions
    - Rational Functions
    - Trigonometric Functions

F.IF.C.7g★ Determine the end behavior of the graph of a polynomial function using the degree and leading coefficient.

- **Clarifications/Examples:**
    - Refer to the wording of the standard.
- **Function families to which this standard applies:**
    - Polynomial Functions

F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- **Clarifications/Examples:**
    - Understand why a particular form of an expression would reveal properties such as zeros, extrema, intercepts, etc.
    - Rewrite rational functions to reveal discontinuities.
- **Function families to which this standard applies:**
    - Logarithmic/Exponential Functions
    - Polynomial Functions
    - Radical Power Functions
    - Rational Functions
    - Trigonometric Functions

F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables or by a verbal description).

- **Clarifications/Examples:**
    - Extend student understanding by using more complex situations.

- **Function families to which this standard applies:**
    - Logarithmic/Exponential Functions
    - Polynomial Functions
    - Radical Power Functions
    - Rational Functions
    - Trigonometric Functions

DOMAIN: BUILDING FUNCTIONS (F.BF)

Cluster: A. Build a function that models a relationship between two quantities.

F.BF.A.1★ Write a function that describes a relationship between two quantities, including more complex functions.

- **Clarifications/Examples:**
    - Include piecewise defined functions comprised of different types of functions.
    - Include composite functions (e.g. $f(x) = \sin(2x)$).
- **Function families to which this standard applies:**
    - Logarithmic/Exponential Functions
    - Polynomial Functions
    - Radical Power Functions
    - Rational Functions
    - Trigonometric Functions

F.BF.A.1c★ Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T\big(h(t)\big)$ is the temperature at the location of the weather balloon as a function of time.

- **Clarifications/Examples:**
    - Identify the domain and range of a composite function.
    - Recognize when a function is the composition of two simpler functions (e.g. $f(x) = e^{\sin x}$).
- **Function families to which this standard applies:**
    - Logarithmic/Exponential Functions
    - Polynomial Functions
    - Radical Power Functions
    - Rational Functions
    - Trigonometric Functions

Cluster B: Build new functions from existing functions.

F.BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

- **Clarifications/Examples:**
  - Identify any function as an even, an odd function or neither given a graphic or algebraic representation.
  - Identify transformations from parent functions.
- **Function families to which this standard applies:**
  - Logarithmic/Exponential Functions
  - Polynomial Functions
  - Trigonometric Functions
  - Radical Power Functions
  - Rational Functions

F.BF.B.4 Find inverse functions.

- **Clarifications/Examples:**
  - Find inverses of polynomials of the form $f(x) = a(x-h)^n + k$.
  - Note: Read the blog, *Inverse Functions: We're Teaching it all Wrong*, before teaching inverses.
- **Function families to which this standard applies:**
  - Logarithmic/Exponential Functions
  - Polynomial Functions
  - Radical Power Functions
  - Rational Functions
  - Trigonometric Functions

F.BF.B.4b Verify by composition that one function is the inverse of another.

- **Clarifications/Examples:**
  - Refer to the wording of the standard.
- **Function families to which this standard applies:**
  - Polynomial Functions
  - Radical Power Functions
  - Rational Functions

F.BF.B.4c Read values of an inverse function from a graph or a table, given that the function has an inverse.

- **Clarifications/Examples:**
  - Given the numeric or graphic representation of an invertible function, produce the graphic and numeric representation of the inverse.
  - Interpret the reflection of a function over the line $y = x$ as a representation of the inverse of a function.

- **Function families to which this standard applies:**
    - Logarithmic/Exponential Functions
    - Polynomial Functions
    - Radical Power Functions
    - Rational Functions
    - Trigonometric Functions

F.BF.B.4d Produce an invertible function from a non-invertible function by restricting the domain.

- **Clarifications/Examples:**
    - Refer to the wording of the standard.
- **Function families to which this standard applies:**
    - Polynomial Functions
    - Radical Power Functions
    - Trigonometric Functions

F.BF.B.4e Build inverse trigonometric functions.

- **Clarifications/Examples:**
    - Build from unit circle.
- **Function families to which this standard applies:**
    - Trigonometric Functions

F.BF.B.5 Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

- **Clarifications/Examples:**
    - Solve exponential equations using logarithms.
    - Discuss relationships between domain, range, and asymptotes.
- **Function families to which this standard applies:**
    - Logarithmic/Exponential Functions

DOMAIN: LINEAR, QUADRATIC, AND EXPONENTIAL MODELS (F.LE)

Cluster: A. Construct and compare linear, quadratic, and exponential models and solve problems.

F.LE.A.4a★ Use properties of logarithms, including both common and natural logarithms, to rewrite and solve exponential models.

- **Clarifications/Examples:**
    - Write logarithmic functions as inverses of exponential functions, including both common and natural logarithms.

- **Function families to which this standard applies:**
  - Logarithmic/Exponential Functions

DOMAIN: TRIGONOMETRIC FUNCTIONS (T.TF)

Cluster: A. Extend the domain of trigonometric functions using the unit circle.

F.TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

- **Clarifications/Examples:**
  - Define the six trigonometric functions in terms of coordinates from the unit circle.
  - Understand the relationship between right triangle trigonometric ratios and trigonometric functions.
  - Evaluate trigonometric functions using reference angles.
- **Function families to which this standard applies:**
  - Trigonometric Functions

F.TF.A.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\frac{\pi}{3}, \frac{\pi}{4}, and\ \frac{\pi}{6}$ and use the unit circle to express the values of sine, cosine, and tangent for $x, \pi + x, and\ 2\pi - x$ in terms of their values for *x*, where *x* is any real number.

- **Clarifications/Examples:**
  - Refer to the wording of the standard.
- **Function families to which this standard applies:**
  - Trigonometric Functions

F.TF.A.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

- **Clarifications/Examples:**
  - Write the domain of all six trigonometric functions.
  - Write the domain of trigonometric functions under transformations (e.g. $y = \tan x$ versus $y = \tan(2x)$).
- **Function families to which this standard applies:**
  - Trigonometric Functions

Cluster: B. Model periodic phenomena with trigonometric functions.

F.TF.B.5★ Choose trigonometric functions to model real world phenomena.

- **Clarifications/Examples:**
  - Graph all six trigonometric functions.

- **Function families to which this standard applies:**
    - Trigonometric Functions

F.TF.B.6 Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

- **Clarifications/Examples:**
    - Refer to the wording of the standard.
- **Function families to which this standard applies:**
    - Trigonometric Functions

F.TF.B.7★ Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology and interpret them in context.

- **Clarifications/Examples:**
    - Refer to the wording of the standard.
- **Function families to which this standard applies:**
    - Trigonometric Functions

Cluster: C. Prove and apply trigonometric identities.

F.TF.C.9.a Use trigonometric identities to rewrite expressions and as a tool when solving trigonometric equations.

- **Clarifications/Examples:**
    - Use the double angle and half angle identities for trigonometric functions to simplify, verify, and solve expressions and equations involving sine, cosine, and tangent.
    - Emphasize double angle identities are used most frequently in Calculus.
    - Prove trigonometric identities, including Pythagorean identities and even and odd identities, using a variety of strategies. Verify identities graphically.
    - Emphasize Pythagorean identities.
- **Function families to which this standard applies:**
    - Trigonometric Functions

Conceptual Category: Geometry

DOMAIN: SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY (G.SRT)

Cluster: D. Apply trigonometry to general triangles.

G.SRT.D.10 Prove the Laws of Sines and Cosines and use them to solve problems.

- **Clarifications/Examples:**
    - Refer to the wording of the standard.

- **Function families to which this standard applies:**
    - Trigonometric Functions

G.SRT.D.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying, resultant forces).

- **Clarifications/Examples:**
    - Refer to the wording of the standard.
- **Function families to which this standard applies:**
    - Trigonometric Functions

Conceptual Category: Statistics

DOMAIN: INTERPRETING CATEGORICAL AND QUANTITATIVE DATA (S.ID)

Cluster: B. Summarize, represent, and interpret data on two categorical and quantitative variables.

S.ID.B.6d★ Fit a function to data represented by a scatterplot; use functions fitted to data to solve problems in the context of the data.

- **Clarifications/Examples:**
    - Determine the best function to represent data on two-quantitative variables by analyzing the context of the data; the behavior of the scatterplot and the fit of the function to the scatterplot.
- **Function families to which this standard applies:**
    - Logarithmic/Exponential Functions
    - Polynomial Functions
    - Radical Power Functions
    - Trigonometric Functions

Logistics Growth

From Algebra I and into Algebra 2, students studied exponential functions and logarithmic functions. Realistically, many situations previously modeled with exponential functions are more accurately modeled with logistic growth models. Why? What are the similarities and differences between an exponential function and a logistic function? When is it more appropriate to use a logistics model? The answers to these questions are the focus of the standards within this topic.

Conceptual Category: Functions

DOMAIN: LINEAR, QUADRATIC, AND EXPONENTIAL MODELS (F.LE)

Cluster: A. Construct and compare linear, quadratic, and exponential models and solve problems.

F.LE.A.1.d Distinguish between situations that can be modeled with exponential functions and logistic functions.

- **Clarifications/Examples:**
  - Refer to the wording of the standard.

Cluster: B. Interpret expressions for functions in terms of the situation they model.

F.LE.B.5a Interpret the parameters in a logistic function in terms of a context.

- **Clarifications/Examples:**
  - Interpret the parameters A, B, and C in expressions of the form $y = \frac{C}{1+Ae^{-Bx}}$, in terms of a context.

F.LE.B.6 Build and interpret logistic functions to model real-world problems.

- **Clarifications/Examples:**
  - Refer to the wording of the standard.

F.LE.B.6a Sketch and analyze the graphs of logistic functions.

- **Clarifications/Examples:**
  - Refer to the wording of the standard.

F.LE.B.6b Compare and contrast the exponential, logarithmic, and logistic models.

- **Clarifications/Examples:**
  - Refer to the wording of the standard.

F.LE.B.6c Apply understanding of logarithmic and logistic functions to solve real-world problems.

- **Clarifications/Examples:**
  - Refer to the wording of the standard.

Precalculus B

Unlike the Precalculus A standards, which are arranged by conceptual category, the Precalculus B standards are arranged by topic. This structure was used as a means to group standards together that apply to a particular topic.

The topics include:

- Complex Numbers
- Vectors and Matrices
- Sequences and Series
- Limits
- Conic Sections
- Parametric Equations
- Polar Coordinates and Equations

Many of the standards included in Precalculus B go beyond the Common Core Mathematics Plus Standards. These standards were added by Maryland educators to define additional mathematics that would provide prerequisite skills and understandings needed for a variety of upper level mathematics courses. Standards added by Maryland educators are identified by underlined standards codes.

Complex Numbers

The standards in this section may be taught in conjunction with a variety of other topics. The work from Engage New York provides one example of how to incorporate the study of complex numbers into a Precalculus curriculum.

Conceptual Category: Number and Quantity

DOMAIN: THE COMPLEX NUMBER SYSTEM (N.CN)

Cluster: A. Perform arithmetic operations with complex numbers.

N.CN.A.3 Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

- **Clarifications/Examples:**
  - Understand the connection between modulus of a complex number and the magnitude of a vector.

Cluster: B. Represent Complex Numbers and their operations on the complex plane.

N.CN.B.4 Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

- **Clarifications/Examples:**
  - Refer to the wording of the standard.

N.CN.B.5 Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.

- **Clarifications/Examples:**
  - Refer to the wording of the standard.

N.CN.B.6 Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

- **Clarifications/Examples:**
  - This could be addressed in a unit on vectors or polar coordinates.

Vectors and Matrices

The study of vectors and matrices provides the foundational skills needed for courses such as Linear Algebra, Differential Equations and a variety of other upper level mathematics courses. Fields such as Data Science, Machine Learning and Engineering make use of the skills and understanding described by this group of standards.

Conceptual Category: Number and Quantity

DOMAIN: VECTOR AND MATRIX QUANTITIES (N.VM)

Cluster: A. Represent and model with vector quantities.

N.VM.A.1 Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes $(\mathbf{e.g.}\,\mathbf{v}, |\mathbf{v}|, ||\mathbf{v}||, \boldsymbol{v})$

- **Clarifications/Examples:**
  - Understand vocabulary and notation associated with the study of vectors (magnitude, direction, scalar, components, unit vector, resultant force).
  - Write position vectors from initial and terminal points.

N.VM.A.2 Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

- **Clarifications/Examples:**
  - Analyze vectors in terms of their horizontal and vertical components.

N.VM.A.3 Solve problems involving velocity and other quantities that can be represented by vectors.

- **Clarifications/Examples:**
  - Model situations involving multiple vectors.
  - Determine resultant vectors and interpret them in context.

Cluster: B. Perform operations on vectors.

N.VM.B.4 Add and subtract vectors.

- **Clarifications/Examples:**
  - Refer to the wording of the standard.

N.VM.B.4.a Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

- **Clarifications/Examples:**
  - Add vectors symbolically and graphically.

N.VM.B.4.b Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

- **Clarifications/Examples:**
  - Use right triangles to derive formulas for the direction and magnitude of a vector.
  - Determine whether a system is in equilibrium.

N.VM.B.4.c Understand vector subtraction $\mathbf{v} - \mathbf{w}$ as $\mathbf{v} + (-\mathbf{w})$, where $-\mathbf{w}$ is the additive inverse of $\mathbf{w}$, with the same magnitude as $\mathbf{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

- **Clarifications/Examples:**
  - Refer to the wording of the standard.

N.VM.B.5 Multiply a vector by a scalar.

- **Clarifications/Examples:**
  - Refer to the wording of the standard.

N.VM.B.5.a Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$

$c\left(v_x, v_y\right) = \left(cv_x, cv_y\right)$.

- **Clarifications/Examples:**
  - Refer to the wording of the standard.

N.VM.B.5.b Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\|c\mathbf{v}\| = |c|\mathbf{v}$. Compute the direction of $c\mathbf{v}$ knowing that when $|c|\mathbf{v} \neq 0$, the direction of $c\mathbf{v}$ is either along $\mathbf{v}$ (for $c > 0$) or against $\mathbf{v}$ (for $c > 0$).

- **Clarifications/Examples:**
  - Refer to the wording of the standard.

N.VM.B.5.c Determine the dot product of two vectors.

- **Clarifications/Examples:**
  - Refer to the wording of the standard.

Cluster: C. Perform operations on matrices and use matrices in applications.

N.VM.C.6 Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

- **Clarifications/Examples:**
  - Refer to the wording of the standard.

N.VM.C.7 Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

- **Clarifications/Examples:**
    - Refer to the wording of the standard.

N.VM.C.8 Add, subtract, and multiply matrices of appropriate dimensions.

- **Clarifications/Examples:**
    - Refer to the wording of the standard.

N.VM.C.9 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

- **Clarifications/Examples:**
    - Refer to the wording of the standard.

N.VM.C.10 Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

- **Clarifications/Examples:**
    - Refer to the wording of the standard.

N.VM.C.11 Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

- **Clarifications/Examples:**
    - Refer to the wording of the standard.

N.VM.C.12 Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

- **Clarifications/Examples:**
    - Include rotations of 90°, 180°, and 270°, and reflections across *x*-axis, *y*-axis, and over the line $y = x$.

Sequences and Series

In earlier grades, students learned about arithmetic and geometric sequences and their relationships to linear and exponential functions, respectively. The study of sequences and series in Precalculus builds on students' understandings of those sequences and extends students' knowledge to include arithmetic and geometric series, both finite and infinite. The study of series is considered to be one of the foundational concepts in the Calculus BC. This group of standards provides the foundational skills needed for the study of series in Calculus.

Note: ***Underlined standards represent standards added by Maryland educators.***

Conceptual Category: Number and Quantity

DOMAIN: OPERATIONS AND ALGEBRAIC THINKING (N.OA)

Cluster: A. Write and interpret numerical expressions.

N.OA.A.1 Use the notation for the factorial of a non-negative integer, $n!$, to evaluate expressions.

- **Clarifications/Examples:**
  - Refer to the wording of the standard.

Conceptual Category: Algebra

DOMAIN: SEEING STRUCTURE IN EXPRESSIONS (A.SSE)

Cluster: B. Write expressions in equivalent forms to solve problems.

A.SSE.B.4★ Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

- **Clarifications/Examples:**
  - Connect prior learning from Algebra 2 to the new learning in Precalculus.
  - Solve real world problems that involve finite series.

A.SSE.B.4.a★ Express the sums in a series using sigma notation.

- **Clarifications/Examples:**
  - Use the algebra rules for finite sums to evaluate expressions written using sigma notation $\left(\text{e.g. Evaluate } \sum_{k=1}^{5} k(k-1)\,.\right)$.
  - Find the partial sums of a series defined using sigma notation.

A.SSE.B.5 Determine the sum, if it exists, of an infinite geometric series.

- **Clarifications/Examples:**
  - Use the sum of an infinite geometric series to express a repeating decimal as a rational number.
  - Solve real world problems that involve infinite series.
  - Use the algebra rules for finite sums to evaluate expressions written using sigma notation $\left(\text{e.g. Evaluate } \sum_{k=1}^{\infty} \frac{1}{k^2}(k-1)\,.\right)$.

DOMAIN: ARITHMETIC WITH POLYNOMIALS AND RATIONAL EXPRESSION (A.APR)

Cluster: C. Use polynomial identities to solve problems.

A.APR.C.5 Know and apply the Binomial Theorem for the expansion of $(\boldsymbol{x}+\boldsymbol{y})^{\boldsymbol{n}}$ in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle.

- **Clarifications/Examples:**
  - Limit to $n < 6$.
  - Limit to binomials with variable coefficient of one or constants less than four.
  - Understand the connection between the Binomial Theorem and an infinite series.

Conceptual Category: Functions

DOMAIN: INTERPRETING FUNCTIONS (F.IF)

Cluster: A. Understand the concept of function and use function notation.

F.IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

- **Clarifications/Examples:**
  - Understand the similarities and differences between linear functions and arithmetic sequences.
  - Understand similarities and differences between exponential functions and geometric sequences.
  - Include sequences that do not have a simple defining equation.

Cluster: C. Analyze functions using different representations.

F.IF.C.10 Describe the behavior of a sequence.

- **Clarifications/Examples:**
    - Generate the terms of a sequence given a formula for the nth term of the sequence.
    - Use the graph of a sequence to intuitively determine if the sequence converges or diverges.
    - Determine if a sequence is increasing, decreasing, or monotonic.

DOMAIN: BUILDING FUNCTIONS (F.BF)

Cluster: C. Build a function that models a relationship between two quantities.

F.BF.A.1★ Write a function that describes a relationship between two quantities.

- **Clarifications/Examples:**
    - Understand the connection between the formula for the general term a_n of a given sequence and the related function that describes the relationship between the term number of the sequence and the value of the term.
    - Graph and analyze the functions that represent a given sequence.

F.BF.A.1.a★ Determine an explicit expression, a recursive process, or steps for calculation from a context.

- **Clarifications/Examples:**
    - Include a variety of sequences that are neither arithmetic nor geometric.

F.BF.A.2★ Write sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

- **Clarifications/Examples:**
    - Construct a formula for the general term a_n of a given sequence.

Limits

Limits of functions is the underpinning of Calculus. Understanding the concept of a limit of a function is the basis to understanding the derivative of a function. In Precalculus, students will use limits to describe behaviors of functions as well as evaluate limits. A proficient understanding of the various function families from Algebra 2 will enable students to critically analyze the limiting behaviors of functions.

Note: ***Underlined standards represent standards added by Maryland educators.***

Conceptual Category: Functions

DOMAIN: INTERPRETING FUNCTIONS (F.IF)

Cluster: A. Understand the concept of function and use function notation.

F.IF.A.2a Understand the concept of limit of a function.

- **Clarifications/Examples:**
    - Understand that the value of a function at a given point may not be the same as the limit as the function approaches the given point.
    - Understand when limits fail to exist.

Cluster: C. Analyze functions using different representations.

F.IF.C.8c Interpret the behavior of the graph of a function using the concept of limits.

- **Clarifications/Examples:**
    - Use limits to reveal asymptotic or unbounded behavior.
    - Use limits to analyze functions for intervals of continuity or points of discontinuity.
    - Identify different types of discontinuities.

F.IF.C.8d Estimate limits algebraically, numerically and graphically.

- **Clarifications/Examples:**
    - Use various algebraic techniques for evaluating limits.
    - Use a table of values to estimate a limit.
    - Use graphs to estimate limits.
    - Find one-sided limits.
    - Understand and use properties of limits.

Topics in Analytic Geometry

Conic Sections

The students have a strong background with some of the conic sections, having studied the parabola extensively in Algebra I and Algebra II and having studied the circle in Geometry. Precalculus extends this knowledge to include the ellipse and the hyperbola. The students revisit the parabola and circle and learn about the new conic sections from a more analytical approach. Understanding the structure of each conic section and its corresponding equation is the crux of this topic. The advancements in graphing technology make exploring the true beauty and application of conic sections possible.

Note: ***Underlined standards represent standards added by Maryland educators.***

Conceptual Category: Algebra

DOMAIN: SEEING STRUCTURE IN EXPRESSIONS (A.SSE)

Cluster: A. Interpret the structure of expressions.

A.SSE.A.2.a Analyze the structure of the general form of a second degree equation, $\boldsymbol{Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0}$**,** to identify the conic section represented by the equation.

- **Clarifications/Examples:**
  - Evaluate the discriminant, $B^2 - 4AC$, of the general form of a second degree equation, $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, to determine if the graph of the equation is a circle, an ellipse, a hyperbola or a parabola.

A.SSE.B.3.d★ Choose and produce an equivalent form of a second degree equation, $Ax^2 + Cy^2 + Dx + Ey + F = 0$, to reveal and explain properties of the conic section represented by the equation.

- **Clarifications/Examples:**
  - Produce the standard form of a second degree equation given the general form.
  - Recognize how the coefficients of the terms transform the conic section.

A.SSE.B.3.e★ Translate between the standard and general form, $Ax^2 + Cy^2 + Dx + Ey + F = 0$, of a second degree equation.

- **Clarifications/Examples:**
  - Identify key features of a conic section.
  - Recognize how the coefficients of the terms transform the conic section.

Conceptual Category: Geometry

DOMAIN: GEOMETRIC MEASUREMENT AND DIMENSION (G.GMD)

Cluster: B. Visualize relationships between two-dimensional and three-dimensional objects.

G.GMD.B.4a Identify the shapes of two-dimensional cross-sections of a right double cone.

- **Clarifications/Examples:**
  - Include special cases of plane/cone intersections that result in degenerate conic sections.

DOMAIN: EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS (G.GPE)

Cluster: A. Translate between the geometric description and the equation for a conic section.

G.GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

- **Clarifications/Examples:**
  - Understand and apply the locus definition for the circle.
  - Connect the geometric definition of the circle to its algebraic equation.
  - Understand the concept of eccentricity of conic sections, and understand the eccentricity of the circle.
  - Use eccentricity to write equations of circles.

G.GPE.A.2 Derive the equation of a parabola given a focus and directrix.

- **Clarifications/Examples:**
  - Include parabolas that have a horizontal axis of symmetry and parabolas that have a vertical axis of symmetry.
  - Derive the standard form of a parabola, centered at the origin.
  - Understand and apply the locus definition for the parabola.
  - Connect the geometric definition of the parabola to its algebraic equation.
  - Understand the concept of eccentricity of conic sections, and understand the eccentricity of the parabola.
  - Use eccentricity to write equations of parabolas.

G.GPE.A.3 Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

- **Clarifications/Examples:**
  - Derive the standard form of an ellipse and a hyperbola, centered at the origin.
  - Understand and apply the locus definitions of the ellipse and the hyperbola.
  - Connect the geometric definitions of the ellipse and hyperbola to their algebraic equations.
  - Understand the concept of eccentricity of conic sections, and understand the eccentricity of the ellipse and the hyperbola.
  - Use eccentricity to write equations of ellipses and hyperbolas.

Parametric Equations

Parametric equations takes a different look at working with a third variable, or in this case, a parameter. Representing direction and motion when curve sketching become possible with the introduction of parametric equations. When students enter into higher level Calculus, parametric equations allow for a deeper look into instantaneous rates of change with respect to motion and time along a curve.

Note: ***<u>Underlined standards represent standards added by Maryland educators.</u>***

Conceptual Category: Parametric Equations

DOMAIN: INTERPRETING PARAMETRIC EQUATIONS (P.IPE)

Cluster: A. Analyze parametric equations.

P.IPE.A.1★ Sketch the curve defined by parametric equations.

- **Clarifications/Examples:**
  - Indicate with an arrow on the curve the direction in which the curve is traced as *t* increases.
  - Describe the motion of a particle with position $(x,\ y)$ as *t* varies in a given interval.

P.IPE.A.2★ Use parametric equations to model and solve motion problems.

- **Clarifications/Examples:**
  - Refer to the wording of the standard.

DOMAIN: CREATING EQUATIONS (P.CED)

Cluster: A. Creating equations that describe numbers or relationships.

P.CED.A.1 Create a single equation, using rectangular coordinates, that is equivalent to a pair of parametric equations.

- **Clarifications/Examples:**
  - Refer to the wording of the standard.

P.CED.A.2 Given a data set, create a parametric equation and a single equation using rectangular coordinates to fit the data.

- **Clarifications/Examples:**
  - Refer to the wording of the standard.

Polar Coordinate System and Polar Equations

Within the students' experience, the study of functions and corresponding graphical representations has strictly been within the *xy*-coordinate plane. Even complex numbers were represented in a rectangular coordinate plane. The study of the polar coordinate system and polar equations puts a new spin on graphing mathematical relationships by introducing radii, directed angles, and rotation. With so much time spent on functions, polar equations provides a refreshing change with the graphing of rose curves, cardioids, lemniscates, limaçons and spirals. Then in Calculus, the students use integration to find the arc length and the area of the petals in these polar curves.

Note: ***Underlined standards represent standards added by Maryland educators.***

Conceptual Category: Geometry

DOMAIN: EXPRESSING GEOMETRIC PROPERTIES WITH EQUATIONS (G.GPE)

Cluster: C. Polar Coordinates.

G.GPE.C.8 Understand the relationship between polar coordinates and Cartesian coordinates.

- **Clarifications/Examples:**
    - Understand that a single point on the polar coordinate plane has more than one set of polar coordinates that can be used to identify the location of the point.

G.GPE.C.9 Convert between polar and rectangular coordinates.

- **Clarifications/Examples:**
    - Refer to the wording of the standard.

G.GPE.C.10 Plot points on a polar coordinate grid.

- **Clarifications/Examples:**
    - Refer to the wording of the standard.

Cluster: D. Polar Equations.

G.GPE.D.11 Convert equations between polar and rectangular forms.

- **Clarifications/Examples:**
    - Refer to the wording of the standard.

G.GPE.D.12★ Graph polar equations by hand and using technology.

- **Clarifications/Examples:**
  - Make connections between the structure of a polar equation and the shape of the corresponding graph.
  - Graphs could include circles, lines, rose curves, cardioids, lemniscates, limaçons and spirals.

G.GPE.D.13 Solve systems of polar equations.

- **Clarifications/Examples:**
  - Restrict solutions to $[0\ ,\ 2\pi)$.