Grade 2 Mathematics

Maryland College and Career Ready Curriculum Framework

Introduction

The Code of Maryland Regulations (COMAR) 13A.04.12.01, Mathematics Instructional Programs for Grades Prekindergarten – 12 states that, "each local education agency shall provide in public schools an instructional program in mathematics each year for all students in grades prekindergarten – 8; Offer in public schools a mathematics program in grades 9—12. Beginning with students entering grade 9 in the 2014—2015 school year, each student shall enroll in a mathematics course in each year of high school that the student attends, up to a maximum of 4 years of attendance, unless in the 5th or 6th year a mathematics course is needed to meet a graduation requirement."

State Frameworks are developed by the Maryland State Department of Education (MSDE) to support local education agencies in providing high-quality instructional programs in mathematics. State Frameworks are defined as supporting documents and provide guidance for implementing the Maryland College and Career Ready Standards for Mathematics which are reviewed and adopted by the Maryland State Board of Education every eight years. State Frameworks also provide consistency in learning expectations for students in mathematics programs across the twenty-four local education agencies as local curriculum is developed and adopted using these documents as a foundation.

MSDE shall update the State Frameworks in Mathematics in the manner and time the State Superintendent of Schools determines is necessary to ensure alignment with best-in-class, research-based practices. Tenure and stability of State Frameworks affords local education agencies the necessary time to procure supporting instructional materials, provide professional development, and to measure student growth within the program. Educators, practitioners, and experts who participate in writing workgroups for State Frameworks represent the diversity of stakeholders across Maryland. State Frameworks in Elementary mathematics grades Prekindergarten – 5 were developed, reviewed, and revised by teams of Maryland educators and practitioners, including local education agency content curriculum specialists, classroom teachers, accessibility staff, and academic researchers and experts in close collaboration with MSDE.

The Grade 2 Mathematics Framework was released in June 2011.

HOW TO READ THE MARYLAND COLLEGE AND CAREER READY CURRICULUM FRAMEWORK

The Maryland College and Career Ready Standards for Mathematics (MCCRSM) at the second-grade level specify the mathematics that all students should study as they prepare to be college and career ready by graduation. The second-grade standards are listed by domains. For further clarification of the standards, reference the appropriate domain in the set of Progression Documents for the Common Core State Standards for Mathematics.

This framework document provides an overview of the Standards that are grouped together to form the domains for grade one. The Standards within each domain are grouped by topic and are in the same order as they appear in the Common Core State Standards for Mathematics. This document is not intended to convey the exact order in which the Standards will be taught, nor the length of time to devote to the study of the different standards.

The framework contains the following:

- **Domains** are intended to convey coherent groupings of content.
- **Clusters** are groups of related standards.
- **Standards** define what students should understand and be able to do.
- **Essential Skills and Knowledge** statements provide language to help teachers develop common understandings and valuable insights into what a student must know and be able to do to demonstrate proficiency with each standard. Maryland mathematics educators thoroughly reviewed the standards and, as needed, provided statements to help teachers comprehend the full intent of each standard.
- **Framework Vocabulary Words** provide definitions of key mathematics vocabulary words found in the document.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. MAKE SENSE OF PROBLEMS AND PERSEVERE IN SOLVING THEM.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. REASON ABSTRACTLY AND QUANTITATIVELY.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. CONSTRUCT VIABLE ARGUMENTS AND CRITIQUE THE REASONING OF OTHERS.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify to improve the arguments.

4. MODEL WITH MATHEMATICS.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. USE APPROPRIATE TOOLS STRATEGICALLY.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. ATTEND TO PRECISION.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. LOOK FOR AND MAKE USE OF STRUCTURE.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. LOOK FOR AND EXPRESS REGULARITY IN REPEATED REASONING.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1)$ equals 3. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO THE MARYLAND COLLEGE AND CAREER READY STANDARDS

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

2.OA Operations and Algebraic Thinking

2.OA.A REPRESENT AND SOLVE PROBLEMS INVOLVING ADDITION AND SUBTRACTION.

2.OA.A.1

Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Essential Skills and Knowledge

- Ability to accurately solve all grade level appropriate one and two step of addition and subtraction word problems (Table 1).
- Ability to solves word problems by demonstrating ability to make sense of problems and persevere in solving them (SMP 1), reason abstractly and quantitatively (SMP 2), makes sense of quantities and their relationships (SMP 4), and models with mathematics (SMP 8).
- Ability to use an empty square or a question mark to represent an unknown in an equation.
- Ability to represent the mathematics in the problem with an equation, where there is only one equal sign.
- Ability to represent the multiple steps in a word problem by recording each step individually using multiple equations (for example: *There are 12 strawberries on the plate. The girls ate 9 of them. Mother put 6 more strawberries on the plate. How many strawberries are there now?* Students record *12-9=3 and then 3+6=8.)*

2.OA.B ADD AND SUBTRACT WITHIN 20.

2.OA.B.2

Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

Essential Skills and Knowledge

- Ability to apply counting strategies to develop automatic recall.
- Ability to use reasoning strategies to make use of known facts (e.g., sums of ten, making ten, doubles, near doubles/inside doubles, doubles plus, counting on).
- Knowledge that subtraction is the inverse of addition (e.g., fact families).

2.OA.C WORK WITH EQUAL GROUPS OF OBJECTS TO GAIN FOUNDATIONS FOR MULTIPLICATION.

2.OA.C.3
Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

Essential Skills and Knowledge

- Ability to use concrete materials to model the meaning of odd and even numbers.
- Knowledge that writing an equation to express an even number as the sum of two equal addends is the same as using doubles (e.g., 4 + 4 = 8, 7 + 7 = 14).
- Ability to skip count by twos.

2.OA.C.4
Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Essential Skills and Knowledge

- Ability to construct rectangular arrays using concrete manipulatives.
- Ability to use repeated addition to find the number of objects in an array.
- Knowledge of rectangular arrays as a foundation for multiplication and a model of the connection between addition and multiplication. At this grade level we are not introducing multiplication, just the understanding of adding multiple sets with the same amount in each set.

2.NBT Number and Operations in Base Ten

2.NBT.A UNDERSTAND PLACE VALUE.

2.NBT.A.1
Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.

Essential Skills and Knowledge

- Ability to use base ten manipulatives (e.g., base ten blocks, Digi-Blocks, stacks of cubes, bundles of sticks, place value arrow cards.
- Knowledge of the value of each place in a number.
- Knowledge of the value of a digit in a specific place.
- Knowledge that the placement of a digit affects the value of that digit.
- See 2NBT1a&b for additional skills and knowledge that are needed for this standard.

2.NBT.A.1a

Understand the following as a special case: 100 can be thought of as a bundle of ten tens -- called a “hundred.”

Essential Skills and Knowledge

- Ability to compose and decompose 100 in a variety of ways lays foundation for regrouping.
- Apply the ability to count by tens.

2.NBT.A.1b

Understand the following as a special case: The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

Essential Skills and Knowledge

- Ability to count by hundreds using place value manipulatives.
- Ability to count by hundreds verbally.

2.NBT.A.2
Count within 1000; skip-count by 5s, 10s, and 100s.

Essential Skills and Knowledge

- Ability to skip count within 100 using the hundreds chart and 1000 using the thousands chart.
- Ability to skip-count starting from various numbers (e.g., counting by tens starting with 27).
- Ability to determine patterns when skip-counting.

2.NBT.A.3
Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

Essential Skills and Knowledge

- Knowledge of the value of digits within a multi-digit number.
- Knowledge of and ability to represent numbers using concrete materials (e.g., base ten blocks, Digi-blocks, place value arrow cards) as well as written numerals and number words.
- Ability to justify the representation with word form and written numerals.

2.NBT.A.4
Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Essential Skills and Knowledge

- Ability to apply place value knowledge to make comparisons (e.g., Look at greatest place value first and compare those digits to see which is greater).

2.NBT.B USE PLACE VALUE UNDERSTANDING AND PROPERTIES OF OPERATIONS TO ADD AND SUBTRACT.

2.NBT.B.5
Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Essential Skills and Knowledge

- Knowledge of addition and subtraction fact families.
- Ability to model regrouping using base ten manipulatives (e.g., base ten blocks, Digi-Blocks, place value arrow cards).
- Knowledge that when regrouping, the value of the number does not change but the place values of the digits within that number change (e.g., When regrouping the problem 324 – 116, 324 becomes 300 + 10 + 14 in order to regroup).

2.NBT.B.6
Add up to four two-digit numbers using strategies based on place value, properties of operations.

Essential Skills and Knowledge

- Knowledge of and ability to apply strategies such as expanded form, empty number line and partial sums.

2.NBT.B.7
Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

Essential Skills and Knowledge

- Represent addition and subtraction three-digit numbers within 1000 using concrete models or drawings and place value strategies, properties of operations, and/or the relationship between addition and subtraction. Then relate the strategy to a written method.
- Demonstrate an understanding of place value when adding or subtraction three-digit numbers.
- Compose or decompose tens or hundreds in order to add or subtraction three-digit numbers.

2.NBT.B.8

Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900.

Essential Skills and Knowledge

- Ability to skip count from a number by 10 and/or 100 including off the decades.
- Ability to model using base ten manipulatives.
- Ability to recognize and use patterns in a thousands chart.

2.NBT.B.9

Explain why addition and subtraction strategies work, using place value and the properties of operations.

Essential Skills and Knowledge

- Ability to use the properties (commutative property for addition, associative property for addition, zero property, identity property) to compute and to support their explanation (Table 2).
- Ability to reason mathematically and explain why their chosen strategy works using words, pictures, and/or symbols to support their explanation.

2.MD Measurement and Data

2.MD.A MEASURE AND ESTIMATE LENGTHS IN STANDARD UNITS.

2.MD.A.1
Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

Essential Skills and Knowledge

- Ability to measure to the nearest inch, centimeter, yard, or meter.
- Knowledge of and ability to explain why we use standard units of measurement instead of non-standard units.
- Ability to estimate before measuring to help determine the appropriate measurement tool and unit.
- Knowledge of the connection between a ruler and a number line.
- Ability to measure real-world objects.

2.MD.A.2
Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

Essential Skills and Knowledge

- Ability to recognize the equivalent units of 12 inches = 1 foot and 100 centimeters = 1 meter as well as non- standard equivalent measurements.

2.MD.A.3
Estimate lengths using units of inches, feet, centimeters, and meters.

Essential Skills and Knowledge

- Ability to use a benchmark when estimating.
- Ability to compare estimates to actual measurements.

2.MD.A.4
Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard-length unit.

Essential Skills and Knowledge

- Ability to connect measurement comparisons to subtraction (comparing) and addition (counting on).

2.MD.B RELATE ADDITION AND SUBTRACTION TO LENGTH.

2.MD.B.5

Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

Essential Skills and Knowledge

- Ability to develop equations to represent word problems.
- Knowledge of inverse relationships.
- Ability to justify the reasonableness of their responses.

2.MD.B.6

Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the number 0, 1, 2, …, and represent whole number sums and differences within 100 on a number line diagram.

Essential Skills and Knowledge

- Ability to locate and represent points on a number line..
- Ability to apply knowledge of anchor points (e.g., 5, 10, 25, 50, 75) as being half-way points between numeral.

2.MD.C WORK WITH TIME AND MONEY.

2.MD.C.7

Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

Essential Skills and Knowledge

- Knowledge of and ability to apply skip counting by 5.
- Knowledge that there are 60 minutes in a hour, 60 seconds in a minute, 24 hours in a day, 12 hours in a.m. and 12 hours in p.m., and know when a.m. and p.m. occur.
- Knowledge of the difference between the minute and hour hands and their purposes.
- Knowledge of concept of quarter- hours and half-hours.
- Knowledge that there are five- minute intervals between each number on the clock face.
- Ability to tell time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.

2.MD.C.8

Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?

Essential Skills and Knowledge

- Ability to identify both sides of currency.
- Ability to count money (dollar bills, quarters, dimes, nickels, and pennies).
- Ability to count mixed sets of currency.
- Ability to count on.
- Knowledge of and ability to apply possible strategies such as drawing pictures, using coins, using a number grid, using a number line, using symbols and/or numbers.

2.MD.D REPRESENT AND INTERPRET DATA.

2.MD.D.9

Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole number units.

Essential Skills and Knowledge

- Understand that a line plot is a representation of data along a number line.
- Ability to identify patterns within the set of data and analyze what the data represents.

2.MD.D.10

Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

Essential Skills and Knowledge

- Ability to collect, sort, organize and graph data.
- Knowledge of the elements of picture graphs and bar graphs.
- Ability to analyze graphs, answer questions about the data, and make decisions based on the data.

2.G Geometry

2.G.A REASON WITH SHAPES AND THEIR ATTRIBUTES.

2.G.A.1

Recognize and draw shapes having specific attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

Essential Skills and Knowledge

- Ability to sort shapes by common attributes.
- Knowledge that plane figures are named by the number of sides.
- Knowledge and investigations include both regular and irregular polygons. (e.g., both equilateral and scalene triangles).

2.G.A.2

Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

Essential Skills and Knowledge

- Ability to partition rectangles into rows and columns of same- size squares lays the foundation for the development of multiplication, area, and fractions.
- Ability to use concrete materials (e.g., color tiles and cubes) to partition a rectangle.
- Ability to apply repeated addition when counting total number of partitions.

2.G.A.3

Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Essential Skills and Knowledge

- Ability to partition circles and rectangles into equal parts lays the foundation for the development of fractions.
- Ability to model using concrete materials (e.g., paper folding, geoboards, fraction manipulatives) to create equal shares.

Grade 2 MD College and Career-Ready Vocabulary

FLUENTLY

Using efficient, flexible and accurate methods for computing.

SUMS OF TEN

Use knowledge of all the whole number pairs that add up to ten to assist in finding other basic fact solutions. Example: If I know that 4 + 6 = 10, then 4 + 8 would equal two more than 10 or 12.

MAKING TEN

When adding 8 + 5, I know that 8 + 2 = 10, so I take 2 from the 5 to make that ten.
Then I have 3 left, so 10 + 3 = 13.

DOUBLES

Is a mental math strategy for addition that is used when adding two addends that are the same number. For example, 2 plus 2 or 3 plus 3, etc. When adding the doubles, the sum is twice as much as one of the addends and it is always an even number.

NEAR DOUBLES

Is a mental math strategy for addition when one addend is one less than the other. For example, if adding 6 + 7, think 6 + 6 = 12 (doubles) and then from the 7 add one more, to equal 13.

INSIDE DOUBLES

Is a mental math strategy for addition when one addend is two more than the other addend. An example is when adding 6 + 8, I can move the 6 one number up to 7 and move the 8 one number back to 7, which gives me the double (inside or between 6 and 8), or 14.

DOUBLES PLUS

Is a mental math strategy for addition using doubles to make known fact and adding on more than one. For example 5 + 9, I know that 5 + 5 = 10, leaving 4 left over. So I add 10 + 4 to get 14. This would also be a sample of using decomposition to solve a problem.

COUNTING ON

An addition counting strategy in which a student starts with one number or set of objects and counts up to solve the problem. For example: Bobby has two counters and Susie has three. How many do they have all together? Bobby says, two and continues to count on by ones to count all of Susie's three counters. Bobby would say 2, 3,4,5. Five is the total number of counters.

RECTANGULAR ARRAYS

Is a rectangular arrangement of objects such as counters, blocks, or graph paper squares placed in rows and columns to represent a multiplication or division equation. For example, place two rows with five stars in each row, arranged in a rectangle equals ten stars altogether. Or 3 rows with 4 blocks in each row, arranged in a rectangle each equals 12 blocks.

PLACE VALUE

The value of a digit as determined by its position in a number. For example: in the number "101" the one is worth either 100 or 1, depending upon its position.

BASE TEN NUMERALS

A base of a numeration system is the number that is raised to various powers to generate the place values of that system. In the base ten numeration system the base is ten. The first place is 10^{0} or 1 (the units place), the second is 10^1 or 10 (the tens place), the third is 10^2 or 100 (the hundreds place), etc. This determines the place value of the different positions in a number.

EXPANDED FORM

A number that is written as the sum of the values of its digits. For example: 7291 = 7000 + 200 + 90 + 1.

PARTIAL SUMS

Involves adding multi-digit numbers by adding parts of the numbers together according to their place value and then adding the partial sums together at the end to get the total. To begin, think of the numbers in expanded form. Many students prefer to start with the largest place value first when adding. For example, adding 234 and 457, are written in expanded form as $200 + 30 + 4$ and $400 + 50 + 7$. Start by adding the hundreds, $200 + 400 = 600$. Then add the tens, $30 + 50 = 80$. Then the $4 + 7 = 11$. Next add the sums of each place value, $600 + 80 + 11 = 691$.

ESTIMATE

Means to give an approximate number or answer.

BENCHMARK

A unit or measurement which serves as a standard by which others may be measured.

Examples: Use the length of an index card as the benchmark to determine the length of a student's desk. Use a yardstick (36 inches) to estimate the width of the classroom.

EQUATION

Is a number sentence stating that the expressions on either side of the equal sign are, in fact equal.

INVERSE OPERATIONS

Is defined as two operations that undo each other. Addition and subtraction are inverse operations. Multiplication and division are inverse operations.

For example, four plus five equals nine and nine subtract four equals five. Also six times five equals thirty and thirty divided by six equals five.

LINE PLOT

Is a graph that shows frequency of data occurrence and is recorded on a number line. The numbers on the number line represent the values of the data being shown. Each data point is represented by a symbol (usually and 'x') above the number on the number line to record its occurrence. For example, If tracking temperatures over 10 days. The x axis would represent the daily temperatures, (from 75 degrees to 85 degrees). An x would be placed above the corresponding temperature to record the day's actual temperature reading. The graph can quickly show the frequency of occurrence of the different temperatures.

PICTURE GRAPH

Is a graph that represents each data item with a picture. Example: On a graph of favorite fruits**,** each student chose a picture of his/her favorite fruit and added it to the graph.

BAR GRAPH

A data display that uses bars to show quantity or numbers so they can be easily compared. For example, the x-axis on the graph would identify the types of animals (dogs, cats, bird, fish). The y-axis on the graph would list the number of pets based on a scale (by 1s, 2,s 5s, etc). A colored bar would show the amount of each type of pet owned by the students in the class.

ANGLES

A figure formed by two rays that have the same endpoint. Types of angles include acute, right, obtuse, and reflex angles. Angles are measured in degrees.

FACES

Are the flat surfaces of a solid figure that does not roll.

TRIANGLES

Are polygons that have only three angles and three sides.

QUADRILATERALS

Are polygons that have only four angles and four sides.

PENTAGONS

Are polygons that have only five angles and five sides.

HEXAGONS

Are polygons that have only six angles and six sides.

CUBES

Are a solid figure that has 6 square faces all equal in size, 8 vertices (corners), and 12 equal edges.

HALVES

A division of a whole or a set into two equal parts.

THIRDS

A division of a whole or a set into three equal parts.

FOURTHS/QUARTERS

A division of a whole or a set into four equal part.

Table 1: Common addition and subtraction situations.

	Results Unknown	Change Unknown	Start Unknown
Add to	Two birds sat on a ledge. Three more birds flew to the ledge. How many birds are now on the ledge? $2 + 3 = ?$	Two birds sat on a ledge. Some more birds flew to the ledge. Then there were five birds on the ledge. How many birds flew over to the first two? $2 + ? = 5$	Some birds sat on a ledge. Three more birds flew to the ledge. Then there were five birds on the ledge. How many birds were on the ledge before? $? + 3 = 5$
Take From	Three oranges were on the table. I ate one orange. How many oranges are on the table now? $3 - 1 = ?$	Three oranges were on the table. I ate some oranges. Then there were two oranges. How many oranges did I eat? $3 - ? = 2$	Some oranges were on the table. I ate one orange. Then there were two oranges. How many oranges were on the table before? $? - 3 = 2$

	Total Unknown	Addend Unknown	Both Addends Unknown
Put Together/ Take Apart	Five red marbles and two green marbles are on the table. How many marbles are on the table? $5 + 2 = ?$	Ten marbles are on the table. Five are red and the rest are green. How many marbles are green? $5 + ? = 10$ or $? + 5 = 10$	Max has five marbles. How many can she put in her left hand and how many in her right hand? $5 = 0 + 5$ $5 = 5 + 0$ $5 = 1 + 4$ $5 = 4 + 1$ $5 = 2 + 3$ $5 = 3 + 2$

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare "more"	**"How many more?" version:** Macy has two cats. Marcus has five cats. How many more cats does Marcus have than Macy? $2 + ? = 5$	**Version with "more":** Marcus has three more cats than Macy. Macy has two cats. How many cats does Marcus have? $2 + 3 = ?$	**Version with "more":** Marcus has three more cats than Macy. Marcus has five cats. How many cats does Macy have? $5 - 3 = ?$
Compare "fewer"	**"How many fewer?" version:** Macy has two cats. Marcus has five cats. How many fewer cats does Macy have than Marcus? $5 - 2 = ?$	**Version with "fewer":** Macy has three fewer cats than Marcus. Macy has two cats. How many cats does Marcus have? $3 + 2 = ?$	**Version with "fewer":** Macy has three fewer cats than Marcus. Marcus has five cats. How many cats does Macy have? $? + 3 = 5$

Darker shading indicates the four Kindergarten problem subtypes. Grade 1 and 2 students work with all subtypes and variants. Unshaded (white) problems are the four difficult subtypes that students should work on in grade 1 but need not master until grade 2.

Adapted from CCSS, p.88, which is based on Mathematics Learning in Early Childhood: Paths Towards Excellence and Equity, National Research Council, 2009, pp. 32-22 and the CCSS Progression document pp. 9.

Table 2: The properties of operations. Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number.

Associative property of addition	(a + b) + c = a + (b + c)
Commutative property of addition	a + b = b + a
Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every *a* there exists -*a* so that a + (-a) = (-a) + a = 0
Associative property of multiplication	(a x b) × c = a × (b × c)
Commutative property of multiplication	a × b = b × a
Multiplicative identity property 1	a × 1 = 1 × a = a
Existence of multiplicative inverses	For every *a* ≠ 0 there exists 1/*a* so that *a* × 1/*a* = 1/*a* × *a* = 1
Distributive property of multiplication over additions	a × (b + c) = a × b + a × c

Adapted from CCSS, p.90.