Grade 3 Mathematics

Maryland College and Career Ready Curriculum Framework

Introduction

The Code of Maryland Regulations (COMAR) 13A.04.12.01, Mathematics Instructional Programs for Grades Prekindergarten – 12 states that, "each local education agency shall provide in public schools an instructional program in mathematics each year for all students in grades prekindergarten – 8; Offer in public schools a mathematics program in grades 9—12. Beginning with students entering grade 9 in the 2014—2015 school year, each student shall enroll in a mathematics course in each year of high school that the student attends, up to a maximum of 4 years of attendance, unless in the 5th or 6th year a mathematics course is needed to meet a graduation requirement."

State Frameworks are developed by the Maryland State Department of Education (MSDE) to support local education agencies in providing high-quality instructional programs in mathematics. State Frameworks are defined as supporting documents and provide guidance for implementing the Maryland College and Career Ready Standards for Mathematics which are reviewed and adopted by the Maryland State Board of Education every eight years. State Frameworks also provide consistency in learning expectations for students in mathematics programs across the twenty-four local education agencies as local curriculum is developed and adopted using these documents as a foundation.

MSDE shall update the State Frameworks in Mathematics in the manner and time the State Superintendent of Schools determines is necessary to ensure alignment with best-in-class, research-based practices. Tenure and stability of State Frameworks affords local education agencies the necessary time to procure supporting instructional materials, provide professional development, and to measure student growth within the program. Educators, practitioners, and experts who participate in writing workgroups for State Frameworks represent the diversity of stakeholders across Maryland. State Frameworks in Elementary mathematics grades Prekindergarten – 5 were developed, reviewed, and revised by teams of Maryland educators and practitioners, including local education agency content curriculum specialists, classroom teachers, accessibility staff, and academic researchers and experts in close collaboration with MSDE.

The Grade 3 Mathematics Framework was released in June 2011.

HOW TO READ THE MARYLAND COLLEGE AND CAREER READY CURRICULUM FRAMEWORK

The Maryland College and Career Ready Standards for Mathematics (MCCRSM) at the third-grade level specify the mathematics that all students should study as they prepare to be college and career ready by graduation. The third-grade standards are listed by domains. For further clarification of the standards, reference the appropriate domain in the set of Progression Documents for the Common Core State Standards for Mathematics or the Grade 3 Mathematics Evidence Statements located on the MSDE MCAP Mathematics webpage.

This framework document provides an overview of the Standards that are grouped together to form the domains for grade one. The Standards within each domain are grouped by topic and are in the same order as they appear in the Common Core State Standards for Mathematics. This document is not intended to convey the exact order in which the Standards will be taught, nor the length of time to devote to the study of the different standards.

The framework contains the following:

- **Domains** are intended to convey coherent groupings of content.
- **Clusters** are groups of related standards.
- **Standards** define what students should understand and be able to do.
- **Essential Skills and Knowledge** statements provide language to help teachers develop common understandings and valuable insights into what a student must know and be able to do to demonstrate proficiency with each standard. Maryland mathematics educators thoroughly reviewed the standards and, as needed, provided statements to help teachers comprehend the full intent of each standard.
- **Framework Vocabulary Words** provide definitions of key mathematics vocabulary words found in the document.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. MAKE SENSE OF PROBLEMS AND PERSEVERE IN SOLVING THEM.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. REASON ABSTRACTLY AND QUANTITATIVELY.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. CONSTRUCT VIABLE ARGUMENTS AND CRITIQUE THE REASONING OF OTHERS.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify to improve the arguments.

4. MODEL WITH MATHEMATICS.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. USE APPROPRIATE TOOLS STRATEGICALLY.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. ATTEND TO PRECISION.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. LOOK FOR AND MAKE USE OF STRUCTURE.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5-3(x-y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. LOOK FOR AND EXPRESS REGULARITY IN REPEATED REASONING.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y-2)/(x-1)$ equals 3. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$, $(x-1)(x^2+x+1)$ and $(x-1)(x^3+x^2+x+1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO THE MARYLAND COLLEGE AND CAREER READY STANDARDS

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

3.OA Operations and Algebraic Thinking

3.OA.A REPRESENT AND SOLVE PROBLEMS INVOLVING MULITPLICATION AND DIVISION.

3.OA.A.1

Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5×7.

Essential Skills and Knowledge

- Knowledge that multiplication is the process of repeated addition and equal groups.
- Interprets a multiplication expression as *x* number of groups of *x* number of objects.
- Ability to use concrete objects, pictures, and arrays to represent the product as the total number of objects.
- Knowledge that the product represented by the array is equivalent to the total of equal addends (2.OA.B.4).
- Ability to apply knowledge of repeated addition up to 5 rows and 5 columns and partitioning, which leads to multiplication (2.OA.B.4).
- Knowledge that the example in Standard 3.0A.A.1 can also represent the total number of objects with 5 items in each of 7 groups (Commutative Property).

3.OA.A.2

Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as* $56 \div 8$.

Essential Skills and Knowledge

- Knowledge that division is the inverse of multiplication and the process of repeated subtraction.
- Ability to use concrete objects to represent the total number and represent how these objects could be shared equally.
- Knowledge that the quotient can either represent the amount in each group or the number of groups with which a total is shared.
- Knowledge that just as multiplication is related to repeated addition, division is related to of repeated subtraction.

3.OA.A.3

Use multiplication and division within 100 to solve word problems in situations involving equal groups, or arrays, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Essential Skills and Knowledge

- Ability to determine when to use multiplication or division to solve a given word problem situation.
- Ability to represent a problem using drawings and equations without or with a symbol for the unknown number.
- Ability to solve different types of multiplication and division word problems (Table 2).

3.OA.A.4

Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = ■ \div 3$, $6 \times 6 = ?$

Essential Skills and Knowledge

- Ability to use concrete objects to compose and decompose sets of numbers.
- Ability to use the inverse operation as it applies to given equation.
- Knowledge of the relationship of multiplication and division as inverse operations.
- Ability to find the unknown in a given multiplication or division equation, where the unknown is represented by a question mark, a box, or a blank line.
- Ability to solve problems that employ different placements for the unknown and product/quotient (Examples: 5 x __= 15, 15 = 3 x __ , 15 ÷ 3 =__, 15 ÷ __= 5, 15 = 5 x ___, ___ = 15 ÷ 3, 3 = __ ÷ 5).

3.OA.B UNDERSTAND PROPERTIES OF MULITPLICATION AND THE RELATIONSHIP BETWEEN MULITPLICATION AND DIVISION.

3.OA.B.5

Apply properties of operations as strategies to multiply and divide. Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2)$ which leads to $40 + 16 = 56$. (Distributive property)

Essential Skills and Knowledge

- Ability to break apart and manipulate the numbers (decomposing and composing numbers).
- Knowledge of the properties of multiplication include Zero, Identity, Commutative, Associative and Distributive properties (Table 3).
- Knowledge that the properties of division include the Distributive Property, but not Commutative or Associative.
- Ability to understand and apply the Properties of Operations as opposed to simply naming them.

- Ability to apply of the Properties of Operations as strategies for increased efficiency.

3.OA.B.6
Understand division as unknown-factor problem. For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.

Essential Skills and Knowledge

- Knowledge that multiplication is the inverse operation of division.
- Ability to apply knowledge of multiplication to solve division problems.

3.OA.C MULTIPLY AND DIVIDE WITHIN 100.

3.OA.C.7
Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Essential Skills and Knowledge

- Knowledge of multiplication and division strategies and properties to achieve efficient recall of facts.
- Ability to use multiple strategies to enhance understanding.
- Ability to model the various properties using concrete materials.

3.OA.D SOLVE PROBLEMS INVOLVING THE FOUR OPERATIONS, AND IDENTIFY AND EXPLAIN PATTERNS IN ARITHMETIC.

3.OA.D.8
Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Essential Skills and Knowledge

- Knowledge of strategies for word problems as established for addition and subtraction (2.OA.A.1).
- Ability to solve word problems that use whole numbers and yield whole-number solutions.
- Ability to determine what a reasonable solution would be prior to solving the word problem.
- Knowledge that a variable refers to an unknown quantity in an equation that can be represented with any letter other than “o”.
- Knowledge that the letter representing a variable takes the place o f an empty box or question mark as used to indicate the unknown in earlier grades.
- Ability to use various strategies applied in one-step word problems to solve multi-step word problems.
- Knowledge of and the ability to use the vocabulary of equation vs. expression.

- Knowledge of and ability to apply estimation strategies, including rounding and front-end estimation, to make sense of the solution(s).
- Ability to apply knowledge of place value to estimation.
- Ability to use critical thinking skills to determine whether an estimate or exact answer is needed in the solution of a word problem.

3.OA.D.9

Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even and explain why 4 times a number can be decomposed into two equal addends.*

Essential Skills and Knowledge

- Ability to apply knowledge of skip counting (1.OA C.5 and 2.NBT.B.2) and explain "why" the pattern works the way it does as it relates to the properties of operations.
- Ability to investigate, discover, and extend number patterns and explain why they work.
- Knowledge that subtraction and division are not commutative.
- Knowledge of multiplication and division properties (Tables 3 and 4).
- Ability to apply knowledge of Properties of operations to explain patterns and why they remain consistent.

3.NBT Number and Operations in Base Ten

3.NBT.A USE PLACE VALUE UNDERSTANDING AND PROPERTIES OF OPERATION TO PERFORM MULTI-DIGIT ARITHMETIC.

3.NBT.A.1

Use place value understanding to round whole numbers to the nearest 10 or 100.

Essential Skills and Knowledge

- Knowledge of place value through 1,000 (2.NBT.A.1) to provide the foundation for rounding whole numbers.
- Knowledge that place value refers to what a digit is worth in a number.
- Knowledge that each place in a number is worth 10 times more than the place to the right of it (e.g., The tens column is worth 10 ones, the hundreds column is worth 10 tens).
- Ability to use a variety of strategies when rounding (e.g., number line, proximity, and hundreds chart).
- Ability to round a three-digit number to the nearest 10 or 100.

3.NBT.A.2

Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Essential Skills and Knowledge

- Knowledge of and ability to apply strategies of decomposing and composing numbers, partial sums, counting up, and counting back by ones, tens, and hundreds.
- Ability to apply alternative algorithms as appropriate.
- Ability to use addition and subtraction interchangeably in computation based on the relationship between the operations.

3.NBT.A.3

Multiply one-digit whole numbers by multiples of 10 in the range of 10-90 (e.g., $9 \times 80, 5 \times 60$) using strategies based on place value and properties of operations.

Essential Skills and Knowledge

- Ability to apply knowledge of place value (e.g., 9 x 80 is 9 times 8 tens = 72 tens).
- Ability to apply the Properties of Operations (Tables 3 and 4).

3.NF Numbers and Operations – Fractions (Limit to fractions with denominators 2, 3, 4, 6, 8)

3.NF.A DEVELOP UNDERSTANDING OF FRACTIONS AS NUMBERS.

3.NF.A.1

Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.

Essential Skills and Knowledge

- Knowledge of the relationship between the number of equal shares and the size of the share (1.G.A.3).
- Knowledge of equal shares of circles and rectangles divided into or partitioned into halves, thirds, and fourths (2.G.A3).
- Knowledge that unit fractions represent 1 of the total number of parts, for example, the fraction is formed by 1 part of a whole which is divided into 4 equal parts.
- Knowledge that a unit fraction can be repeated to make other fractions, for example, one fourth plus one fourth plus one forth equals three fourths.
- Knowledge of the terms numerator (the number of parts being counted) and denominator (the total number of equal parts in the whole).
- Knowledge of and ability to explain and write fractions that represent one whole (e.g., four fourths equals one whole, eight eighths equals one whole).

- Ability to identify and create fractions of a region and of a set, including the use of concrete materials.
- Knowledge of the size or quantity of the original whole when working with fractional parts.

3.NF.A.2

Understand a fraction as a number on the number line; represent fractions on a number line diagram.

3.NF.A.2a

Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.

3.NF.A.2b

Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths of $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.

Essential Skills and Knowledge

- Ability to apply knowledge of whole numbers on a number line to the understanding of fractions on a number line.
- Ability to apply knowledge of unit fractions to represent and compute fractions on a number line.
- Ability to use linear models (e.g., equivalency table and manipulatives such as fraction strips, fraction towers, Cuisenaire rods) for fraction placement on a number line.
- Knowledge of the relationship between the use of a ruler in measurement to the use of a ruler as a number line.
- Knowledge that a number line does not have to start at zero.
- Ability to identify fractions on a number line with tick marks as well as on number lines without tick marks.

3.NF.A.3

Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

Essential Skills and Knowledge

- Ability to use concrete manipulatives and visual models to explain reasoning about fractions.
- Knowledge that equivalent fractions are ways of describing the same amount by using different-sized fractional parts. (e.g., $\frac{1}{2}$ is the same as $\frac{2}{4}$ or $\frac{3}{6}$ or $\frac{4}{8}$).
- Ability to use a variety of models when investigating equivalent fractions (e.g., number line, Cuisenaire rods, fraction towers, fraction circles, equivalence table, fraction strips).
- Ability to relate equivalency to fractions of a region or fractions of a set.
- Ability to use benchmarks of 0, 1/2, and 1 comparing fractions.

- Knowledge of and experience with fractional number sense to lay foundation for manipulating, comparing, finding equivalent fractions, etc.

3.NF.A.3a

Represent two fractions as equivalent (equal) if they are the same size, or the same point on the number line.

Essential Skills and Knowledge

- Ability to describe the same amount by using different-sized fractional parts. (e.g., $\frac{1}{2}$ is the same as $\frac{2}{4}$ or $\frac{3}{6}$ or $\frac{4}{8}$).
- Ability to use number lines as well as fractions of a set or fractions of a region to model equivalent fractions.
- Ability to use a variety of models to investigate relationships of equivalency.
- Ability to recognize 1 whole is represented by a fraction with the same numerator and denominator ($\frac{3}{3}$ or $\frac{4}{4}$, etc.) since they are the same size and same point on the number line.

3.NF.A.3b

Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2}=\frac{2}{4}$, $\frac{4}{6}=\frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

Essential Skills and Knowledge

- Ability to describe the same amount by using different-sized fractional parts (e.g., $\frac{1}{2}$ is the same as $\frac{2}{4}$ or $\frac{3}{6}$ or $\frac{4}{8}$).
- Ability to use fraction models (e.g., fraction towers, fraction strips) to justify understanding of equivalent fractions.

3.NF.A.3c

Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3=\frac{3}{1}$; recognize that $6=\frac{6}{1}$; locate $\frac{4}{4}$ and 1 at the same point of a number line diagram.

Essential Skills and Knowledge

- Knowledge of the denominator as the number of parts that a whole is divided into in order to explain why a denominator of 1 indicates whole number.

3.NF.A.3d

Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Essential Skills and Knowledge

- Ability to use benchmarks of 0, $\frac{1}{2}$, and 1 to explain relative value of fractions.
- Knowledge that as the denominator increases the size of the part decreases.
- Knowledge that when comparing fractions, the whole must be the same size.
- Ability to use a variety of models when comparing fractions (e.g., number line, and manipulatives such as Cuisenaire rods, fraction towers, fraction strips).

3.MD Measurement and Data

3.MD.A SOLVE PROBLEMS INVOLVING MEASUREMENT AND ESTIMATION OF INTERVALS OF TIME, LIQUID VOLUMES, AND MASSES OF OBJECTS.

3.MD.A.1
Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

Essential Skills and Knowledge

- Ability to tell time to the nearest 5- minute interval (2.MD.C.7).
- Ability to tell time to the nearest minute in a.m. and p.m.
- Ability to measure time intervals in minutes.
- Ability to solve time problems by using the number line model as opposed to an algorithm.
- Ability to initially add minutes in order to find the end time followed by working backwards to find start time.
- Ability to find the elapsed time of an event.
- Ability to relate fractions and time ($\frac{1}{4}$ with quarter hour, $\frac{1}{2}$ with half past the hour).
- Ability to find start time, end time, or elapsed time.

3.MD.A.2

Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.

Essential Skills and Knowledge

- Participates in multiple hands-on experiences to understand the quantity of grams, kilograms, and liters, as well as how they compare with each other.
- Ability to use the tools to measure mass and volume.
- Ability to explain the differences between mass and volume.
- Ability to solve one step word problems involving masses or volumes that are given in the same units using drawings with understanding.
- Ability to solve one-step word problems involving masses or volumes using the four operations.

3.MD.B REPRESENT AND INTERPRET DATA.

3.MD.B.3

Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how may less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

Essential Skills and Knowledge

- Knowledge that the use of "square" is referring to interval on the scale and that not all graphs will include a "square" but all graphs should include intervals.
- Ability to apply experience with constructing and analyzing simple, single-unit scaled bar and picture graphs (pictograph) with no more than 4 categories (2.MD.D.10).
- Knowledge of increased scale and intervals (moving to graphs representing more than one item and the intervals representing 2, 5, 10 on the graph, etc.) and expanding to one-step and two-step problem-solving with given data.
- Knowledge that the interval of scale is the amount from one tick mark to the next along the axis and that the scale would be determined based on the values being represented in the data.
- Knowledge of and ability to connect understanding of locating points on a number line with locating points between intervals on a given axis. (e.g., if given a scale counting by 5s students would need to be able to estimate the location of 13 between intervals of 10 and 15.
- Ability to apply the information in the Key when interpreting fractions of a symbol on a picture graph.

3.MD.B.4
Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot where the horizontal scale is marked off in appropriate units – whole numbers, halves, or quarters.

Essential Skills and Knowledge

- Ability to apply prior experience with the measurement of lengths being marked and recorded on line plots to the nearest whole unit (3.NF.A.2) .

3.MD.C GEOMETRIC MEASUREMENT: UNDERSTAND CONCEPTS OF AREA AND RELATE AREA TO MULTIPLICATION AND TO ADDITION.

3.MD.C.5
Recognize area as an attribute of plane figures and understand concept of area measurement.

Essential Skills and Knowledge

- Ability to apply experience with partitioning rectangles into rows and columns to count the squares within (2.OA.C.4).
- Knowledge that area is the measure of total square units inside a region or how many square units it takes to cover a region.

3.MD.C.5a
A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

Essential Skills and Knowledge

- Ability to use square units of measure (inch tile) to measure figures and identify length, perimeter, or area to give the total measure.

3.MD.C.5b
A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

Essential Skills and Knowledge

- Ability to use square units of measure to cover a variety of plane figures without gaps or overlaps to provide the total area of the figure.

3.MD.C.6
Measure areas by counting unit squares (square cm, square m, square in., square ft., and improvised units).

Essential Skills and Knowledge

- Ability to use manipulatives and visual models to calculate area.

3.MD.C.7

Relate area to the operations of multiplication and addition.

Essential Skills and Knowledge

- Ability to explain the relationship of multiplication arrays and area (3.OA.A.3).

3.MD.C.7a

Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

Essential Skills and Knowledge

- Ability to justify the understanding of area by comparing tiling and counting with repeated addition/multiplication.

3.MD.C.7b

Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

Essential Skills and Knowledge

- Ability to apply the formula for area of a rectangle to solve word problems.
- Ability to apply the formula for area when the measurement of one side is not given.
- Use understanding of area to identify false reasoning and explain how to correctly find the area of rectangles.

3.MD.C.7c

Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

Essential Skills and Knowledge

- Ability to construct rectangles on grid paper and decompose them by cutting them up or color coding them to investigate area.
- Ability to use a pictorial model of the distributive property to solve area word problems.
- Knowledge that, for example, when working with a rectangle with side lengths of 7 units by 8 units, let *a* represent 7 and *b + c* represent a decomposition of 8 (e.g. 5 + 3, 6 + 2, 4 + 4, 7 + 1, etc.). In other words, 7 × 8 is the same as (7 × 2) + (7 × 6).

3.MD.C.7d

Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.

Essential Skills and Knowledge

- This is an extension of 3.MD.C.7c.
- Knowledge that rectilinear figures refer to any polygon with all right angles.
- Ability to apply knowledge of finding area of a single polygon to finding areas of two non-overlapping rectangles to find the area f the whole figure.
- Ability to apply knowledge of area for one rectangle to finding the area of rectilinear figures when the measurement for one side is missing.

3.MD.D GEOMETRIC MEASUREMENT: RECOGNIZE PERIMETER AS AN ATTRIBUTE OF PLANE FIGURES AND DISTINGUISH BETWEEN LINEAR AND AREA MEASURES

3.MD.D.8

Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Essential Skills and Knowledge

- Knowledge that the perimeter is the distance around a region.
- Ability to use manipulatives and visual models to find the perimeter of a polygon.
- Ability to apply a variety of strategies to find the perimeter of a polygon.
- Ability to explain and model the relationships between area and perimeter using concrete materials (e.g., color tiles and geoboards).
- Use understanding of perimeter to identify false reasoning and explain how to correctly find the perimeter of plane figures.
- Knowledge that this is a geometry application of unit fractions (3.NF.A.1) and ability to make use of unit fraction understanding.
- Ability to use concrete materials to divide shapes into equal areas (e.g., pattern blocks , color tiles, geoboards).

3.G Geometry

3.G.A REASON WITH SHAPES AND THEIR ATTRIBUTES.

3.G.A.1

Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Essential Skills and Knowledge

- Ability to compare and sort polygons based on their attributes, extending beyond the number of sides (2.G.A.1).
- Ability to explain why two polygons are alike or why they are different based on their attributes.

3.G.A.2

Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as $\frac{1}{4}$ of the area of the shape.

Essential Skills and Knowledge

- Knowledge that this is a geometry application of unit fractions (3.NF.A.1) and ability to make use of unit fraction understanding.
- Ability to use concrete materials to divide shapes into equal areas (e.g., pattern blocks, color tiles, geoboards).

Grade 3 MD College and Career-Ready Vocabulary

PRODUCT

The result (answer) when two numbers are multiplied. Example: 5 x 4 = 20 and 20 is the product.

PARTITIONING

To divide the whole into equal parts

QUOTIENT

The number resulting from dividing one number by another

SHARE

A unit or equal part of a whole.

ARRAYS

A rectangular arrangement of counters, blocks, or graph paper square in rows and columns to represent a multiplication or division equation. For example, two rows of four blocks equals eight blocks or three rows of four buttons equals twelve buttons.

MEASUREMENT QUANTITIES

Units that are used to measure, include inches, feet, pints, quarts, centimeters, meters, liters, square units, etc.

INVERSE OPERATION

Two operations that undo each other. Addition and subtraction are inverse operations.
Multiplication and division are inverse operations. Examples: 4 + 5 = 9; 9 – 5 = 4 or
6 x 5 = 30; 30 ÷ 5 = 6

DECOMPOSING

breaking a number into two or more parts to make it easier to compute .Example: When combining a set of 5 and a set of 8, a student might decompose 8 into a set of 3 and a set of 5, making it easier to see that the two sets of 5 make 10 and then there are 3 more for a total of 13.

COMPOSING

Composing (opposite of decomposing) is the process of joining numbers into a whole number…to combine smaller parts. Examples: 1 + 4 = 5; 2 + 3 = 5. These are two different ways to "compose" 5. Or What are all the numbers that can make up 6?

PROPERTIES OF OPERATIONS

The properties of operations apply to the rational number system, the real number system, and the complex number system. The properties of operations are listed below

- **Zero Property*:*** In addition, any number added to zero equals that number. Example: 8 + 0 = 8 In multiplication, any number multiplied by zero equals zero. Example: 8 x 0 = 0
- **Identity Property*:*** In addition, any number added to zero equals that number. Example: 8 + 0 = 8. In multiplication, any number multiplied by one equals that number. Example: 8 x 1 = 8
- **Commutative Property*:*** In both addition and multiplication, changing the order of the factors when adding or multiplying will not change the sum or the product. Example: 2 + 3 = 5 and 3 + 2 = 5; 3 x 7 = 21 and 7 x 3 = 21
- **Associative Property*:*** in addition and multiplication, changing the grouping of the elements being added or multiplied will not change the sum or product. Examples: (2 + 3) + 7 = 12 and 2 + (3 + 7) = 12; (2 x 3) x 5 = 30 and 2 x (3 x 5) =30
- **Distributive Property*:*** a property that relates two operations on numbers, usually multiplication and addition or multiplication and subtraction. This property gets its name because it 'distributes' the factor outside the parentheses over the two terms within the parentheses. For example, to multiply 2 times 11, decompose the 11 into 7 + 4. First multiply- parenthesis, two times seven, close parenthesis, plus parenthesis, two times four ,close parenthesis. Which gives you the results of fourteen plus eight which equals twenty-two.

FLUENCY

Using efficient, flexible and accurate methods of computing

PLACE VALUE

the value of a digit as determined by its position in a number. For example: in the number "101" the one is worth either 100 or 1, depending upon its position.

BASE TEN NUMERALS

a base of a numeration system is the number that is raised to various powers to generate the place values of that system. In the base ten numeration system the base is ten. The first place is 10^0 or 1 (the units place), the second is 10^1 or 10 (the tens place), the third is 10^2 or 100 (the hundreds place), etc. This determines the place value of the different positions in a number.

VARIABLE

A letter or other symbol that represents a number. A variable need not represent one specific number; it can stand for many different values. Examples: 2 x ? = 16 and a + 6 = b.

EQUATION

Is a number sentence stating that the expressions on either side of the equal sign are in fact equal.

EXPRESSION

Is a mathematical phrase that may contain numbers, a variable constants, grouping symbols and basic operational symbols (add, subtract, multiply, and divide). They do not contain equal signs or symbols of comparison (greater than/ less than symbols)

ESTIMATION STRATEGIES

To estimate is to give an approximate number or answer. Some possible strategies used to estimate are:

- **Front End Estimation**: This method takes its name from the way you will round the number. Instead of rounding a number to a given place value, we round whatever the digit is in the very front of the number. We use that number and all the other numbers become zeros. For example in the number 356, the three is in the 'front' of the number (or the largest place value).
- **Rounding**: This method involves identifying a place value location to round to, for example round this number to the nearest tenor nearest hundred. If the number you are rounding is followed by a 5,6,7,8, or 9 you round up to the nearest ten or hundred. If the number you are rounding is followed by 0,1,2,3,4, round the number down. For example to round to the nearest ten for the number 37 because the number ends in 7, you would round up to 40. If the number is 32, you would round down to 30 because the number ended in a 2.
- **Compatible Numbers**: This method of estimation is used when asked to find the sum, difference, product or quotient. It involves making a number so that it is easy to mentally compute. We can do this by rounding the numbers to the nearest ten, twenty, fifty, or hundred. An example, adding three hundred and sixty-six and four hundred twenty-three, you could change three hundred sixty six to three hundred sixty and four hundred twenty three to four hundred twenty to compute the answer of seven hundred eighty.

WHOLE

Working with fractions, the whole refers to the entire region, set, or line segment which is divided into equal parts or segments.

NUMERATOR

The number above the line in a fraction; names the number of parts of the whole being referenced. For example: I ate 3 pieces of a pie that had 6 pieces in all. So, 3 out of 6 parts of a whole is three sixths ($\frac{3}{6}$). The three is the numerator, the part I ate. The six is the denominator or the total number of pieces of pie.

DENOMINATOR

Is the number below the line in a fraction; states the total number of parts in the whole. For example, I ate three pieces of pie that had six pieces in all. So, 3 out of 6 parts of a whole is three sixths ($\frac{3}{6}$). The three is the numerator, the part I ate. The six is the denominator or the total number of pieces of pie.

FRACTION OF A REGION

A fraction is the number of shaded parts of a region (rectangle) that has been partitioned into equal parts. For example, if a rectangle has been partitioned into six equal parts and 2 of the parts are shaded, the fraction two sixths would represent the shaded part of the rectangle (region).

UNIT FRACTION

A fraction with a numerator of one. For Example, one half, one third, on sixth.

LINEAR MODELS

Are used to represent fractions, perform operations with fractions, and identify their placement on a number line. Some examples are fraction strips, fraction towers, Cuisenaire rods, number line and equivalency tables.

EQUIVALENT FRACTIONS

Equivalent fractions have different numerators and denominators, but the same value. For example, one half of a region is partitioned into two equal pieces and with one of the two pieces shaded. Three sixths is shown by partitioning a region into six equal pieces, three of which are shaded. Both fractions have one half of the area shaded.

BENCHMARK FRACTIONS

Are fractions that are commonly used for estimation or for comparing fractions, usually one half or one whole. When comparing fractions the comparison is only valid when comparing the two fractions with the same whole.

MIXED NUMBER

A number that is made up of a whole number and a fraction.

SCALED PICTURE GRAPH

Is a graph constructed using repetition of a single picture or symbol to represent the various categories of data. It includes a scale which explains how many data items are represented by the single graphic. For example on a graph of favorite fruit there are four selections of fruit- apples, grapes, pears, and peaches. There is a picture of fruit which represents two pieces of fruit. On the graph there are four pictures of fruit which means eight children selected apples as their favorite fruit.

SCALED BAR GRAPH

A graph that shows the relationship among data by using bars to represent quantities within each category of data. Example: A graph about Pets has an x axis with four choices of pets- dogs, cats, birds, fish. The y axis provides a scale for the number of pets ranges by a scale of 0,2,4,6, etc to 14. There are four vertical bars that vary in length for each pet and tell the number of each pet owned by the students. There are 12 dogs, 8 cats, 2 birds, and 5 fish.

LINE PLOT

Is a graph that shows frequency of data occurring along a number line. Line plots provide a quick way to organize and interpret data. They are best used when comparing fewer than 25 different numbers. An example of a line plot could be showing frequency of temperatures over a two week period. The number line can show temperature degrees between 75 and 90 (each tick mark would represent a degree of temperature). Daily temperatures are entered by making an 'x' in the column that correlates to the temperature on the line plot. At the end of two weeks it is easy to see the range of temperatures and frequency of temperatures.

AREA

The number of square units needed to cover a flat region

TILING

Highlighting the square units on each side of a rectangle (length and width) to show its relationship to multiplication and that by multiplying the side lengths, the area can be determined. For example, a rectangle with side with of 7 square units in length and 3 square units in width will have an area of 21 square units once the region inside of the side lengths are filled in.

RECTILINEAR FIGURES

A polygon which has only 90 degrees (only right angles) and possible 270 degree angles and an even number of sides. The figure can be decomposed into smaller rectangles using a horizontal or vertical line to find the area of the smaller rectangles. The areas of the two smaller rectangles can be added together to find the area of the larger figures.

Table 1: Common addition and subtraction situations.

	Results Unknown	Change Unknown	Start Unknown
Add to	Two birds sat on a ledge. Three more birds flew to the ledge. How many birds are now on the ledge? $2 + 3 = ?$	Two birds sat on a ledge. Some more birds flew to the ledge. Then there were five birds on the ledge. How many birds flew over to the first two? $2 + ? = 5$	Some birds sat on a ledge. Three more birds flew to the ledge. Then there were five birds on the ledge. How many birds were on the ledge before? $? + 3 = 5$
Take From	Three oranges were on the table. I ate one orange. How many oranges are on the table now? $3 - 1 = ?$	Three oranges were on the table. I ate some oranges. Then there were two oranges. How many oranges did I eat? $3 - ? = 2$	Some oranges were on the table. I ate one orange. Then there were two oranges. How many oranges were on the table before? $? - 3 = 2$

	Total Unknown	Addend Unknown	Both Addends Unknown
Put Together/ Take Apart	Five red marbles and two green marbles are on the table. How many marbles are on the table? $5 + 2 = ?$	Ten marbles are on the table. Five are red and the rest are green. How many marbles are green? $5 + ? = 10$ or $? + 5 = 10$	Max has five marbles. How many can she put in her left hand and how many in her right hand? $5 = 0 + 5$ $5 = 5 + 0$ $5 = 1 + 4$ $5 = 4 + 1$ $5 = 2 + 3$ $5 = 3 + 2$

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare "more"	**"How many more?" version:** Macy has two cats. Marcus has five cats. How many more cats does Marcus have than Macy? $2 + ? = 5$	**Version with "more":** Marcus has three more cats than Macy. Macy has two cats. How many cats does Marcus have? $2 + 3 = ?$	**Version with "more":** Marcus has three more cats than Macy. Marcus has five cats. How many cats does Macy have? $5 - 3 = ?$
Compare "fewer"	**"How many fewer?" version:** Macy has two cats. Marcus has five cats. How many fewer cats does Macy have than Marcus? $5 - 2 = ?$	**Version with "fewer":** Macy has three fewer cats than Marcus. Macy has two cats. How many cats does Marcus have? $3 + 2 = ?$	**Version with "fewer":** Macy has three fewer cats than Marcus. Marcus has five cats. How many cats does Macy have? $? + 3 = 5$

Darker shading indicates the four Kindergarten problem subtypes. Grade 1 and 2 students work with all subtypes and variants. Unshaded (white) problems are the four difficult subtypes that students should work on in grade 1 but need not master until grade 2.

Adapted from CCSS, p.88, which is based on Mathematics Learning in Early Childhood: Paths Towards Excellence and Equity, National Research Council, 2009, pp. 32-22 and the CCSS Progression document pp. 9.

Table 2: Common multiplication and division situations.

Problem Situation	Unknown Product 3 × 6 = ?	Group Size Unknown (How many in each group?) 3 × ? = 18 and 18 ÷ 3 = ?	Number of Groups Unknown (How many groups?)
Equal groups (Grades 3 – 5)	There are 3 bags with 6 plums in each bag. How many plums are there in all? Measurement example: Alex needs 3 lengths of string, each 6 inches long. How much string will Alex need?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? Measurement example: Dale has 18 inches of string, which he will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? Measurement example: Ruth has 18 inches of string, which she will cut into pieces that are 6 inches long. How many pieces of string will she have?
Arrays and Area (Grades 3 – 5)	There are 3 rows of apples with 6 in each row. How many apples are there? OR The apples in the grocery window are in 3 rows and 6 columns. How many apples are there? Area Example: What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? OR If 18 apples are arranged into an array with 3 rows, how many columns of apples are there? Area example: A rectangle has an area of 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? OR If 18 apples are arranged into an array with 3 columns, how many rows are there? Area example: A rectangle has an area of 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare (Grades 4 – 5) Multiplicative Compare problems appear first in Grade 4, with the “times as much” language.	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example: A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? Measurement example: A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? Measurement example: A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?

Adapted from CCSS, p.89.

Table 3: The properties of operations. Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number.

Associative property of addition	(a + b) + c = a + (b + c)
Commutative property of addition	a + b = b + a
Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every *a* there exists -*a* so that a + (-a) = (-a) + a = 0
Associative property of multiplication	(a x b) × c = a × (b × c)
Commutative property of multiplication	a × b = b × a
Multiplicative identity property 1	a × 1 = 1 × a = a
Existence of multiplicative inverses	For every *a* ≠ 0 there exists 1/*a* so that *a* × 1/*a* = 1/*a* × *a* = 1
Distributive property of multiplication over additions	a × (b + c) = a × b + a × c

Adapted from CCSS, p.90.

Table 4: The properties of equality. Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	a = a
Symmetric property of equality	If a = b, then b = a.
Transitive property of equality	If a = b and b = c, then a = c.
Addition property of equality	If a = b, then a + c = b + c.
Subtraction property of equality	If a = b, then a – c = b – c.
Multiplication property of equality	If a = b, then a × c = b × c.
Division property of equality	If a = b and c ≠ 0, then a ÷ c = b ÷ c.
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Adapted from CCSS, p.90.