Grade 5 Mathematics

Maryland College and Career Ready Curriculum Framework

Introduction

The Code of Maryland Regulations (COMAR) 13A.04.12.01, Mathematics Instructional Programs for Grades Prekindergarten – 12 states that, "each local education agency shall provide in public schools an instructional program in mathematics each year for all students in grades prekindergarten – 8; Offer in public schools a mathematics program in grades 9—12. Beginning with students entering grade 9 in the 2014—2015 school year, each student shall enroll in a mathematics course in each year of high school that the student attends, up to a maximum of 4 years of attendance, unless in the 5th or 6th year a mathematics course is needed to meet a graduation requirement."

State Frameworks are developed by the Maryland State Department of Education (MSDE) to support local education agencies in providing high-quality instructional programs in mathematics. State Frameworks are defined as supporting documents and provide guidance for implementing the Maryland College and Career Ready Standards for Mathematics which are reviewed and adopted by the Maryland State Board of Education every eight years. State Frameworks also provide consistency in learning expectations for students in mathematics programs across the twenty-four local education agencies as local curriculum is developed and adopted using these documents as a foundation.

MSDE shall update the State Frameworks in Mathematics in the manner and time the State Superintendent of Schools determines is necessary to ensure alignment with best-in-class, research-based practices. Tenure and stability of State Frameworks affords local education agencies the necessary time to procure supporting instructional materials, provide professional development, and to measure student growth within the program. Educators, practitioners, and experts who participate in writing workgroups for State Frameworks represent the diversity of stakeholders across Maryland. State Frameworks in Elementary mathematics grades Prekindergarten – 5 were developed, reviewed, and revised by teams of Maryland educators and practitioners, including local education agency content curriculum specialists, classroom teachers, accessibility staff, and academic researchers and experts in close collaboration with MSDE.

The Grade 5 Mathematics Framework was released in June 2011.

HOW TO READ THE MARYLAND COLLEGE AND CAREER READY CURRICULUM FRAMEWORK

The Maryland College and Career Ready Standards for Mathematics (MCCRSM) at the fifth-grade level specify the mathematics that all students should study as they prepare to be college and career ready by graduation. The fifth-grade standards are listed by domains. For further clarification of the standards, reference the appropriate domain in the set of Progression Documents for the Common Core State Standards for Mathematics or the Grade 5 Mathematics Evidence Statements located on the MSDE MCAP Mathematics webpage.

This framework document provides an overview of the Standards that are grouped together to form the domains for grade one. The Standards within each domain are grouped by topic and are in the same order as they appear in the Common Core State Standards for Mathematics. This document is not intended to convey the exact order in which the Standards will be taught, nor the length of time to devote to the study of the different standards.

The framework contains the following:

- **Domains** are intended to convey coherent groupings of content.
- **Clusters** are groups of related standards.
- **Standards** define what students should understand and be able to do.
- **Essential Skills and Knowledge** statements provide language to help teachers develop common understandings and valuable insights into what a student must know and be able to do to demonstrate proficiency with each standard. Maryland mathematics educators thoroughly reviewed the standards and, as needed, provided statements to help teachers comprehend the full intent of each standard.
- **Framework Vocabulary Words** provide definitions of key mathematics vocabulary words found in the document.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. MAKE SENSE OF PROBLEMS AND PERSEVERE IN SOLVING THEM.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. REASON ABSTRACTLY AND QUANTITATIVELY.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. CONSTRUCT VIABLE ARGUMENTS AND CRITIQUE THE REASONING OF OTHERS.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify to improve the arguments.

4. MODEL WITH MATHEMATICS.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. USE APPROPRIATE TOOLS STRATEGICALLY.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. ATTEND TO PRECISION.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. LOOK FOR AND MAKE USE OF STRUCTURE.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. LOOK FOR AND EXPRESS REGULARITY IN REPEATED REASONING.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1)$ equals 3. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO THE MARYLAND COLLEGE AND CAREER READY STANDARDS

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

5.OA Operations and Algebraic Thinking

5.OA.A WRITE AND INTERPRET NUMERICAL EXPRESSIONS.

5.OA.A.1
Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

Essential Skills and Knowledge

- Ability to build on knowledge of order of operations to find the value of an expression without variables.

5.OA.A.2
Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*

Essential Skills and Knowledge

- Knowledge of the term, "expression," and the difference between this term and equation.
- Ability to interpret calculation into numerical terms and numerical expressions into words.
- Ability to write simple expressions without actually calculating them.
- Ability to apply their reasoning of the four operations and knowledge of place value to describe the relationship between numbers.
- This standard does not include variables, only numbers and operational signs.
- Ability to apply their understanding of four operations and grouping symbols to write expressions.

5.OA.B ANALYZE PATTERNS AND RELATIONSHIPS.

5.OA.B.3
Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Essential Skills and Knowledge

- Ability to generate and analyze patterns (4.OA.C.5).
- Knowledge that corresponding terms are used to create ordered pairs .
- Ability to apply knowledge of the coordinate system. Graphing points in the first quadrant of a coordinate plane (5.G.A.1 and 5.G.A.2).

5.NBT Number and Operations in Base Ten

5.NBT.AUNDERSTAND THE PLACE VALUE SYSTEM.

5.NBT.A.1
Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.

Essential Skills and Knowledge

- Ability to identify place value of individual digits in a multi-digit whole number (4.NBT.A.1 and 4.NBT.A.2)
- Ability to describe the relationship between decimal fractions and decimal notation (4.NBT.C.5, 4.NBT.C.6, and 4.NBT.C.7) .
    - Note: Grade 4 to hundredths and grade 5 to thousandths.
- Ability to identify and describe the integration of decimal fractions into place value system.
- Ability to reason about the magnitude of whole numbers, decimals, and decimal fractions (e.g., identify which digit is 10 times, 100 times, or $\frac{1}{10}, \frac{1}{100}$ etc. of another digit).

5.NBT.A.2
Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole number exponents to denote powers of 10.

Essential Skills and Knowledge

- Knowledge of exponents with powers of 10.
- Knowledge of when dividing by powers of 10, the exponent above the 10 indicates how many places the decimal point is moving (how many times we are dividing by 10, the number because ten time smaller). When dividing by 10, the decimal point moves to the left.

5.NBT.A.3
Read, write, and compare decimals to the thousandths.

> **5.NBT.A.3a**
> Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g. $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (\frac{1}{10}) + 9 \times (\frac{1}{100}) + 2 \times (\frac{1}{1000})$.
>
> **5.NBT.A.3b**
> Compare two decimals to the thousandths based on meanings of the digits in each place, using >, =, < symbols to record the results of comparisons.

Essential Skills and Knowledge

- See the skills and knowledge that are stated in the standard.

5.NBT.A.4
Use place value understanding to round decimals to any place.

Essential Skills and Knowledge

- Ability to reason and explain the answers to a problem that requires rounding using knowledge of place value and number sense.
- Ability to identify two possible answers and use understanding of place value to compare the given number to the possible answers to round to any place.
- Ability to use benchmark numbers to compare and round numbers to any place.

5.NBT.B PERFORM OPERATIONS WITH MULTI-DIGIT WHOLE NUMBERS AND WITH DECIMALS TO HUNDREDTHS.

5.NBT.B.5
Fluently multiply multi-digit whole numbers using the standard algorithm.

Essential Skills and Knowledge

- Accurate recall of single digit multiplication facts .
- Select and use accurate and efficient methods to compute such as mental math, properties of multiplication, decomposing/composing numbers, (as they transition to the standard algorithm).
- Ability to apply an understanding of place value and multiplying multi-digit numbers.
- Ability to use the standard algorithm and recognize the importance of place value.

5.NBT.B.6
Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Essential Skills and Knowledge

- Accurate recall of division facts and multiplication facts.
- Ability to use a variety of strategies to find whole number quotients such as, relationship between multiplication and division, properties of operations, place value, etc.
- Ability to apply place value understanding to multiplying and dividing multi-digit numbers
- Ability to explain calculations by using equations or models.
- Ability to identify from the problem context the meaning of the divisor (size of groups or number of groups).
- Limits up to four-digit and two –digit divisors, can include remainders.

5.NBT.B.7
Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Essential Skills and Knowledge

- Ability to use concrete models and pictorial representations to perform operations with decimals to hundredths.
- Ability to recognize that the product is not always larger than its factors.
- Ability to recognize that the quotient is not always smaller than the dividend.
- Ability to write numerical expressions or equations to represent the problem and solution.
- Ability to reason and explain how the models, pictures, or strategies were used to solve the problem.

5.NF Number and Operations – Fractions

5.NF.A USE EQUIVALENT FRACTIONS AS A STRATEGY TO ADD AND SUBTRACT FRACTIONS.

5.NF.A.1

Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,* $\frac{2}{3}+\frac{5}{4}=\frac{8}{12}+\frac{15}{12}=\frac{23}{12}$. *In general,* $\frac{a}{b}+\frac{c}{d}=\frac{(ad+bc)}{bd}$.

Essential Skills and Knowledge

- Knowledge of understanding of addition and subtraction of fractions with like denominators and unit fractions from grade 4 (4.NF.B.3.a-d).
- Ability to find the common denominator by finding the product of the denominators using visual fraction models.
- Ability to create equivalent fractions for each addend by using the **identity property**.

5.NF.A.2

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result* $\frac{2}{5}+\frac{1}{2}=\frac{3}{7}$ *by observing that* $\frac{3}{7}<\frac{1}{2}$.

Essential Skills and Knowledge

- Knowledge of understanding addition and subtraction of fractions as joining and separating parts referring to the same whole (4.NF.A 3a).
- Ability to add and subtract fractions with unlike denominators by using visual fraction models or equations (5.NF.A.1).
- Ability to use benchmark fractions to estimate mentally and assess the reasonableness of answers.
- Ability to explain why their answer is reasonable using benchmark fractions and fraction sense.

5.NF.B APPLY AND EXTEND PREVIOUS UNDERSTANDINGS OF MULTIPLICATION AND DIVISION TO MULTIPLY AND DIVIDE FRACTIONS.

5.NF.B.3

Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret $\frac{3}{4}$ as the result of dividing 3 by 4, noting that $\frac{3}{4}$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $\frac{3}{4}$. If 9 people want to share a 50- pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

Essential Skills and Knowledge

- Ability to recognize that a fraction is a representation of division.
- Relate division of whole numbers to division of fractions.

5.NF.B.4

Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

5.NF.B.4a

Interpret the product $\frac{a}{b} \times q$ as *a* parts of *a* partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations *a* × *q* ÷ *b*. *For example, use a visual fraction model to show $\frac{2}{3} \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$. In general, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.*

Essential Skills and Knowledge

- Ability to apply knowledge of multiplication of a whole numbers to multiplying fractions by a whole number as repeated addition of a unit fraction using fraction models and then equations (4.NF.B.4c). For example: $3 \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$.
- Ability to create a story context to multiply a whole number by a fraction.
- Ability to multiply a fraction times a fraction using fraction models and then equations.
- Ability to create a story context to multiply a fraction by a fraction.

5.NF.B.4b

Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas.

Essential Skills and Knowledge

- Knowledge of unit fractions to multiply all fractions (4.NF.B.3a-b).
- Knowledge of using rectangular arrays to find area (4.NBT.B.5).

5.NF.B.5

Interpret multiplication as scaling (sizing) by:

5.NF.B.5a

Comparing the size of a product to the size of one factor based on the size of the other factor, without performing the indicated multiplication.

Essential Skills and Knowledge

- Ability to apply knowledge of multiplicative comparisons with whole numbers (4.OA.A.1) to comparisons with fractions.
- Ability to reason abstractly about the magnitude of products with fractions. For example: 5 × 3 is 5 times as big as 3 (grade 4) and $\frac{1}{2} \times 3$ is half the size of 3 (Grade 5).

5.NF.B.5b

Explaining why multiplying a given number by a fraction greater than a 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $\frac{a}{b} = \frac{n \times a}{n \times b}$ to the effect of multiplying $\frac{a}{b}$ by 1.

Essential Skills and Knowledge

- Ability to understand the differences in results when multiplying whole numbers and when multiplying fractions. (when multiplying a fraction greater than 1, the number increases and when multiplying by a fraction less than 1, the number decreases)
- Ability to explore the concepts in this standard while doing work on 5.NF.B.4a

5.NF.B.6

Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Essential Skills and Knowledge

- Ability to apply the concepts and knowledge of this cluster.
- Solve a variety of problems using multiplication of fractions. Include problems involving fraction by a whole number, fraction by a fraction, fraction by a mixed number, and mixed number by a mixed number.

5.NF.B.7

Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. Interpret division of a unit fraction.

Essential Skills and Knowledge

- Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But **division of a fraction by a fraction is not a requirement at this grade**.
- Dividing unit fractions by a whole number is an introductory concept in grade 5.
- Ability to determine the amount of unit fractions in a whole.
- Ability to understand multiplication and division as equal groups or equal shares and the number of objects in each group or share.

5.NF.B.7a

Interpret division of a unit fraction by a non-zero whole number and compute such quotients. *For example, create a story context for* $\frac{1}{3} \div 4$ *and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $\frac{1}{3} \div 4 = \frac{1}{12}$ *because* $\frac{1}{12} \times 4 = \frac{1}{3}$.

Essential Skills and Knowledge

- Ability to demonstrate word problems when unit fractions are divided by a non-zero whole number and compute the quotient using visual fraction models. For example: Maria has 1/4 of a pizza to share with 3 people. How much of the pizza will each person receive?
- Ability to explain the relationship between multiplication and division of a unit fraction by a non-zero whole number.

5.NF.B.7b

Interpret division of a whole number by a unit fraction and compute such quotients. *For example, create a story context for* $4 \div \frac{1}{5}$ *and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $4 \div \frac{1}{5} = 20$ *because* $20 \times \frac{1}{5} = 4$.

Essential Skills and Knowledge

- Ability to demonstrate word problems when a whole number is divided by unit fraction and compute the quotient using visual fraction models.
- Create a story context for a whole number divided by a unit fraction and compute the quotient for the problem.
- Example: Create a word problem for 6 ÷ 1/8. Solve the problem and use the relationship between multiplication and division to explain your problem.

5.NF.B.7c

Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share* $\frac{1}{2}$ *lb. of chocolate equally? How many* $\frac{1}{3}$*-cup servings are in 2 cups of raisins?*

Essential Skills and Knowledge

- Knowledge of the relationship between multiplication and division. (4.NBT.B.6., 5.NF.B.7a, and 7b).
- Ability to apply all the concepts from this cluster to solve a variety of word problems involving division of a fraction by a whole number or whole number by a fraction.
- Ability to explain the relationship between multiplication and division of fractions by a whole number.

5.MD Measurement and Data

5.MD.A CONVERT LIKE MEASUREMENT UNITS WITHIN A GIVEN MEASUREMENT SYSTEM.

5.MD.A.1
Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m) and use these conversions in solving multi-step problems.

Essential Skills and Knowledge

- Ability to convert measurements within the same measurement system (Metric and Customary) (4.MD.A.1).
- Ability to distinguish between and select the appropriate units for length, mass, and volume. (4.MD.A.1).
- Knowledge of the relationships between the units of measurement in both metric and customary systems.
- Ability to use appropriate measuring tools for both systems (yardsticks/meter sticks, rulers (metric and customary) Measuring cups, etc.
- Ability to convert measurements found within the context of multi-step, real world word problems. (4.MD.A.2).
- Ability to use knowledge of base ten system to converting metric measurements for length, mass, and volume.

5.MD.B REPRESENT AND INTERPRET DATA.

5.MD.B.2
Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$). Use operations for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

Essential Skills and Knowledge

- Knowledge of whole numbers on a line plot to represent and interpret fractional data on a line plot (4.MD.B.4).
- Ability to use equivalent fractions to add and subtract fractions (5.NF.A.1 and 5.NF.A.2).
- Ability to apply knowledge of multiplication and division to multiply and divide fractions based on the line plot data.
- Ability to measure objects to the nearest $\frac{1}{2}, \frac{1}{4}$, or $\frac{1}{8}$ and display that data on a line plot. Use operations to solve problems based on the data. Or interpret data on a line plot to use operations to solve problems.

5.MD.C GEOMETRIC MEASUREMENT: UNDERSTAND CONCEPTS OF VOLUME AND RELATE VOLUME TO MULITPLICATION AND TO ADDITION.

5.MD.C.3

Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

5.MD.C.3a

A cube with side length of one unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

5.MD.C.3b

A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

Essential Skills and Knowledge

- Ability to understand that volume introduces a third dimension to figures.
- Ability to understand the differences in liquid volume and solid volume. Liquid volume (4.MD.A.2) fills the three-dimensional space of a container and takes the shape of the container. Solid volume fills the space of a three-dimensional container by packing the container with solid units.
- Knowledge that solid units are composed of 1 unit by 1 unit by 1 unit. This solid unit is called a cubic unit and it is the standard measure for finding volume.
- Ability to find the total volume of a solid figure by packing a solid figure with cubic inches or centimeters, without gaps or overlaps, and then counting the cubic units to find the total volume (Does not 'fill' a container randomly with cubic units).

5.MD.C.4

Measure volumes by counting cubes, using cubic cm. cubic in., cubic ft., and improvised units.

Essential Skills and Knowledge

- Ability to pack unit cubes without gaps or overlaps into right rectangular prisms and count the cubes to determine the volume.
- Understand that the volume of a right rectangular prism can be found by first packing the container with cubes to making rows to create layers (an array of rows and columns that pack the container without gaps or overlaps).
- Understand that given the total volume of a right rectangular prism, be able to decompose it, understanding it can be partitioned into layers, each layer can be decomposed into rows and each row into cubes. Compare the results to other right rectangular prisms with different dimensions.
- Conceptualize a layer as the unit that is composed of rows, each row is composed of individual units. The layers are iterated to be able to predict the number of cubes needed to fill a box when the net of a box is given.

5.MD.C.5

Relate volume to the operations of multiplication and addition, and solve real-world and mathematical problems involving volume.

5.MD.C.5a

Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

Essential Skills and Knowledge

- Apply multiplicative reasoning to determine volume, looking for and making use of previously learned structure for finding volume using unit cubes in iterated layers. Use multiplication to find the area of the base of the container (length and width = 1 layer) and multiply the resulting area by the number of layers (height).
- Ability to understand the height of the prism tells how many layers will fit in the prism.
- Ability to represent threefold whole-number product as volume to represent the associative property (Volume is a derived attribute once length is specified; it can be computed as the product of three length measurements or as the product of one area and one length).

5.MD.C.5b

Apply the formulas $V = (l)(w)(h)$ and $V = (b)(h)$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.

Essential Skills and Knowledge

- Ability to apply knowledge of finding volume in previous standards to develop the formula for finding volume (length × width × height).
- Ability to be given a total volume to find the dimensions of the figure.
- Ability to find the volume of a figure with an unknown dimension.

5.MD.C.5c
Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non- overlapping parts, applying this technique to solve real-world problems.

Essential Skills and Knowledge

- Ability to find the total volume of two non-overlapping right rectangular prisms by finding the volume of each right rectangular prism and adding the volumes of both.
- Solve problems with two non-overlapping right rectangular prisms.

5.G Geometry

5.G.A GRAPH POINTS ON THE COORDINATE PLANE TO SOLVE REAL-WORLD AND MATHEMATICAL PROBLEMS.

5.G.A.1

Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y- axis and y-coordinate).

Essential Skills and Knowledge

- See the skills and knowledge that are stated in the standard.

5.G.A.2
Represent real-world mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Essential Skills and Knowledge

- See the skills and knowledge that are stated in the standard.

5.G.B CLASSIFY TWO-DIMENSIONAL FIGURES IN A HIERARCHY BASED ON PROPERTIES.

5.G.B.3

Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

Essential Skills and Knowledge

- Knowledge of classifying two dimensional figures; see relationships among the attributes of two-dimensional figures.

5.G.B.4

Classify two-dimensional figures in a hierarchy based on properties.

Essential Skills and Knowledge

- See the skills and knowledge that are stated in the standard.

Grade 5 MD College and Career-Ready Vocabulary

PARENTHESES

( ) A grouping symbol. This is commonly used as the first grouping symbol. They indicate in which order the operations should be preformed.

BRACKETS

[ ] A grouping symbol. Brackets are used when a second grouping symbol is needed.

BRACES

{ } A grouping symbol. Braces are used when a third grouping symbol is needed. Grouping symbols are often used in order of operation problems. Example:

$$4 + 2\{3 + 1[5(3 + 2)] + 6\}$$
$$4 + 2\{3 + 1[5 \times 6] + 6\}$$
$$4 + 2\{3 + 1 \times 30 + 6\}$$
$$4 + 2\{3 + 30 + 6\}$$
$$4 + 2\{39\}$$
$$4 + 78 = 82$$

EXPRESSIONS

A mathematical phrase that has no equality or inequality signs Example:

7×5 $\quad$ $2(3 + 4)$ $\quad$ $3 - 9 + 7$ $\quad$ $2 + 4 - 6 \times 8$

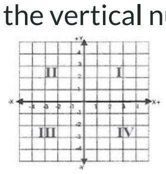
CORRESPONDING TERMS

Given a rule, generate a number pattern with two sets. For example, given one rule to multiply the first number by 2 to get the second number and given a second rule to multiply the first number by 4 to get the second number. The second number in second rule should be twice the size of the second number in the first rule.

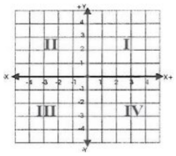
First rule:	(1,2)	(2,4)	(3,6)	(4,8)…
	↕	↕	↕	↕
Second rule:	(1,4)	(2,8)	(3,12)	(4,16)…

COORDINATE PLANE

This is a coordinate plan. It can be called a coordinate grid or Cartesian plane. It has two axes and four quadrants. The two number lines form the axes. The horizontal number line is called the x-axis and the vertical number line is called the y-axis. The four quadrants are numbered counterclockwise.

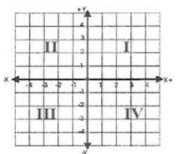


The two number lines form the axes. The horizontal number line is called the x-axis and the vertical number line is called the y-axis.



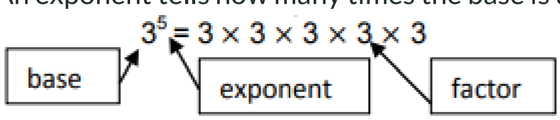
QUADRANT

The four quadrants are numbered in a counter-clockwise direction. They are formed by the x-axis and y-axis.



EXPONENT

An exponent tells how many times the base is used as a factor.



IDENTITY PROPERTY OF MULTIPLICATION

This is also called the multiplicative identity. The identity for multiplication is the number 1, because 1 multiplied by any number is equal to that number. m × 1 = 1 × m is equal to m.

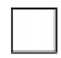
BENCHMARK FRACTIONS

Benchmark fractions are used for estimation. When you add $\frac{1}{3}+\frac{3}{5}=\frac{5}{15}+\frac{9}{15}$ you get $\frac{14}{15}$. When you estimate the addition, you would think that $\frac{1}{3}$ is closer to $\frac{1}{2}$ and $\frac{3}{5}$ is closer is $\frac{1}{2}$ so your estimated answer would be about 1. The benchmarks used with fractions are 0, $\frac{1}{2}$, 1. Also $\frac{1}{3}$ is less than $\frac{1}{2}$ and $\frac{3}{5}$ is more than $\frac{1}{2}$, so we know that $\frac{1}{3} < \frac{3}{5}$.

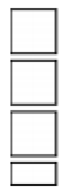
UNIT FRACTIONS TO MULTIPLY ALL FRACTIONS

Refer to 4.NF.4 for background knowledge. A model for $4 \times 3\frac{1}{2}$ is shown.

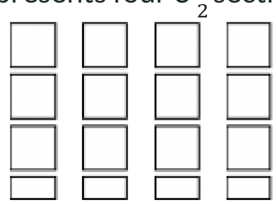
Represents one whole.



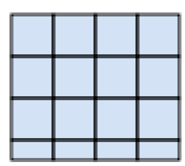
Represents $3\frac{1}{2}$



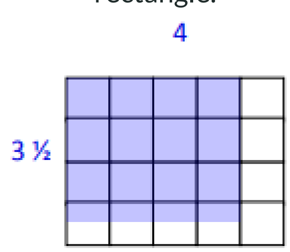
Represents four $3\frac{1}{2}$ sections.



Connect these to form an array.



To find the area of a rectangle you must multiply the length and width of the rectangle. In the rectangle shown you would multiply $3\frac{1}{2} \times 4 = 14$. Or you could count the tiles that are covered in the rectangle.

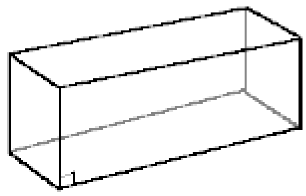


REAL WORLD PROBLEMS

A problem that has context not just symbols. It is a practical world problem as opposed to an academic world problem. They are drawn from actual events or situations.

RIGHT RECTANGULAR PRISM

A solid figure that has all right angles with opposite faces congruent rectangles.



HIERARCHY

Any system of things ranked one above the another. Example: A hierarchy for geometry would be simple closed curves, polygons, quadrilaterals, parallelograms.

Table 1: Common addition and subtraction situations.

	Results Unknown	Change Unknown	Start Unknown
Add to	Two birds sat on a ledge. Three more birds flew to the ledge. How many birds are now on the ledge? $2 + 3 = ?$	Two birds sat on a ledge. Some more birds flew to the ledge. Then there were five birds on the ledge. How many birds flew over to the first two? $2 + ? = 5$	Some birds sat on a ledge. Three more birds flew to the ledge. Then there were five birds on the ledge. How many birds were on the ledge before? $? + 3 = 5$
Take From	Three oranges were on the table. I ate one orange. How many oranges are on the table now? $3 - 1 = ?$	Three oranges were on the table. I ate some oranges. Then there were two oranges. How many oranges did I eat? $3 - ? = 2$	Some oranges were on the table. I ate one orange. Then there were two oranges. How many oranges were on the table before? $? - 3 = 2$

	Total Unknown	Addend Unknown	Both Addends Unknown
Put Together / Take Apart	Five red marbles and two green marbles are on the table. How many marbles are on the table? $5 + 2 = ?$	Ten marbles are on the table. Five are red and the rest are green. How many marbles are green? $5 + ? = 10$ or $? + 5 = 10$	Max has five marbles. How many can she put in her left hand and how many in her right hand? $5 = 0 + 5$ $5 = 5 + 0$ $5 = 1 + 4$ $5 = 4 + 1$ $5 = 2 + 3$ $5 = 3 + 2$

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare "more"	**"How many more?" version:** Macy has two cats. Marcus has five cats. How many more cats does Marcus have than Macy? $2 + ? = 5$	**Version with "more":** Marcus has three more cats than Macy. Macy has two cats. How many cats does Marcus have? $2 + 3 = ?$	**Version with "more":** Marcus has three more cats than Macy. Marcus has five cats. How many cats does Macy have? $5 - 3 = ?$
Compare "fewer"	**"How many fewer?" version:** Macy has two cats. Marcus has five cats. How many fewer cats does Macy have than Marcus? $5 - 2 = ?$	**Version with "fewer":** Macy has three fewer cats than Marcus. Macy has two cats. How many cats does Marcus have? $3 + 2 = ?$	**Version with "fewer":** Macy has three fewer cats than Marcus. Marcus has five cats. How many cats does Macy have? $? + 3 = 5$

Darker shading indicates the four Kindergarten problem subtypes. Grade 1 and 2 students work with all subtypes and variants. Unshaded (white) problems are the four difficult subtypes that students should work on in grade 1 but need not master until grade 2.

Adapted from CCSS, p.88, which is based on Mathematics Learning in Early Childhood: Paths Towards Excellence and Equity, National Research Council, 2009, pp. 32-22 and the CCSS Progression document pp. 9.

Table 2: Common multiplication and division situations.

Problem Situation	Unknown Product 3 × 6 = ?	Group Size Unknown (How many in each group?) 3 × ? = 18 and 18 ÷ 3 = ?	Number of Groups Unknown (How many groups?)
Equal groups (Grades 3 – 5)	There are 3 bags with 6 plums in each bag. How many plums are there in all? Measurement example: Alex needs 3 lengths of string, each 6 inches long. How much string will Alex need?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? Measurement example: Dale has 18 inches of string, which he will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? Measurement example: Ruth has 18 inches of string, which she will cut into pieces that are 6 inches long. How many pieces of string will she have?
Arrays and Area (Grades 3 – 5)	There are 3 rows of apples with 6 in each row. How many apples are there? OR The apples in the grocery window are in 3 rows and 6 columns. How many apples are there? Area Example: What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? OR If 18 apples are arranged into an array with 3 rows, how many columns of apples are there? Area example: A rectangle has an area of 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? OR If 18 apples are arranged into an array with 3 columns, how many rows are there? Area example: A rectangle has an area of 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare (Grades 4 – 5) Multiplicative Compare problems appear first in Grade 4, with the "times as much" language.	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? Measurement example: A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? Measurement example: A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? Measurement example: A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?

Adapted from CCSS, p.89.

Table 3: The properties of operations. Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number.

Associative property of addition	(a + b) + c = a + (b + c)
Commutative property of addition	a + b = b + a
Additive identity property of 0	a + 0 = 0 + a = a
Existence of additive inverses	For every *a* there exists -*a* so that a + (-a) = (-a) + a = 0
Associative property of multiplication	(a x b) × c = a × (b × c)
Commutative property of multiplication	a × b = b × a
Multiplicative identity property 1	a × 1 = 1 × a = a
Existence of multiplicative inverses	For every *a* ≠ 0 there exists 1/*a* so that *a* × 1/*a* = 1/*a* × *a* = 1
Distributive property of multiplication over additions	a × (b + c) = a × b + a × c

Adapted from CCSS, p.90.

Table 4: The properties of equality. Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	a = a
Symmetric property of equality	If a = b, then b = a.
Transitive property of equality	If a = b and b = c, then a = c.
Addition property of equality	If a = b, then a + c = b + c.
Subtraction property of equality	If a = b, then a – c = b – c.
Multiplication property of equality	If a = b, then a × c = b × c.
Division property of equality	If a = b and c ≠ 0, then a ÷ c = b ÷ c.
Substitution property of equality	If a = b, then b may be substituted for a in any expression containing a.

Adapted from CCSS, p.90.