# MARYLAND COLLEGE AND CAREER READY STANDARDS

## FOR STATISTICS AND PROBABILITY

Maryland State Department of Education
STATE OF MARYLAND | 200 WEST BALTIMORE STREET

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## Overview

In an effort to increase focus and coherence in both Algebra I and Algebra II, some of the Maryland College and Career Ready Mathematics Standards (MCCRMS) from the Statistics and Probability conceptual category were removed from those courses. The Statistics Standards that remain within Algebra I and Algebra II include S.ID.B.5 – S.ID.C.8. In order to allow more time for in-depth study of all of the MCCRMS for Statistics and Probability, Maryland's Local School Systems (LSS) are encouraged to offer a stand-alone Statistics and Probability course developed with the standards outlined in this document.

The Maryland College and Career Ready Statistics and Probability standards define what students should understand and be able to do in their study of Statistics and Probability. The standards included in this document represent the essential skills and understandings that will enable a student to be successful in an Advanced Placement Statistics course or in an introductory credit-bearing college-level statistics course.

The incorporation of technology is an essential component to consider when developing a course aligned to the Statistics and Probability standards included in this document. Modern statistics is based in technology, and many software tools and resources are freely available to enhance student understanding. Instruction in Statistics and Probability courses should move beyond formulas and calculations and should aim for a greater focus on the interpretation, comparisons and proper use of statistics.

**Additional Consideration**

The work associated with the First in the World Maryland Mathematics Reform Initiative (FITW MMRI) resulted in the development of a mathematics pathway offered by Maryland's Institutions of Higher Education (IHE) that includes Statistics as the first credit-bearing college-level mathematics course available to students attending Maryland's IHEs. Students may be required to take a mathematics placement test to determine if they are prepared to take this entry level Statistics course upon entering one of Maryland's IHEs. The work associated with the FITW MMRI included the development of a non-credit course entitled Mathematical Foundations for students determined as not possessing the prerequisite mathematics skills needed for success in the entry level Statistics course. The content of this course may be accessed using the provided hyperlink. Maryland LSS's course developers may consider including some of the skills from the Mathematical Foundations course into locally developed Statistics and Probability courses in an effort to reduce the need for students to take non-credit mathematics coursework upon entering one of Maryland's IHEs.

Additional resources to use for the development of a Statistics and Probability course include the Mathematics Foundations for Success in Introductory Statistics from the Dana Center as well as the Pre-K – 12 report from the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report published by the American Statistical Association.

## Course Sequences

As with all Maryland standards, the Statistics and Probability standards reflected in this document do not represent a curriculum and do not suggest pedagogy. The Statistics and Probability standards are intended to serve as guidance to Maryland districts as they develop statistics curricular documents. The course developed with this framework should be offered to students after they have earned credits for both Algebra I and Geometry. As of 2021, Algebra II is still a required course for admission in to the colleges in the University of Maryland System. Specific decisions about when the Statistics and Probability course is offered will be made by each local school system.

## Understanding the Statistics and Probability Standards Document

The standards tables include:

- Standards with Clarifications/Examples

  Clarifications provide details about the standards. Clarifications and examples are not provided for standards where added clarity is not needed. Examples and pedagogical suggestions are also provided, where appropriate.

- Plus Standards

  The Maryland College and Career Ready Standards for Mathematics include standards referred to as "plus standards." These are labeled with a plus sign (+). They are not included in Algebra I, Geometry or Algebra II as these standards represent mathematics needed for advanced level mathematics coursework. All of the plus standards related to Statistics and Probability have been included in this framework. The plus sign has been removed in the name of the standard because it is considered a necessary part of any statistics and probability course developed using this framework.

- Coherence

  A section has been added to this framework to showcase what prior knowledge students will bring in connection to the standards listed as well as a connection to what students will see as they progress through these standards. Many of these coherence connections relate back to middle school statistics standards, but they also show the interconnectedness of the standards within this framework.

- Standards for Mathematical Practice

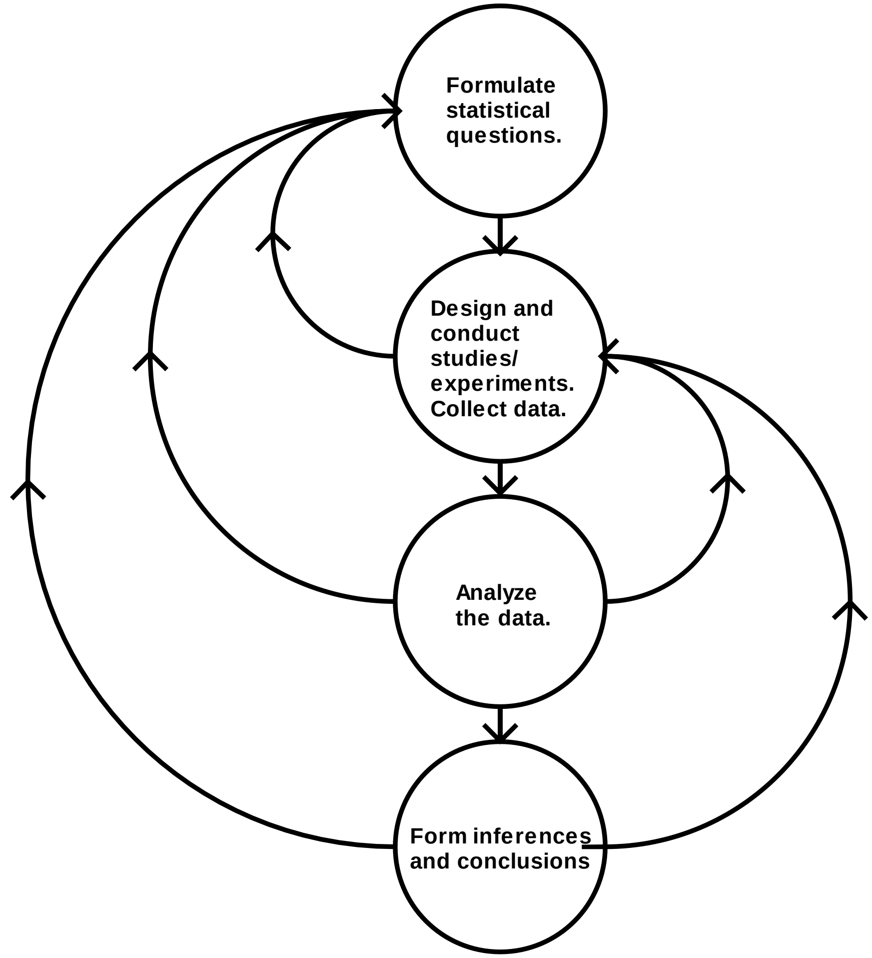
The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## The Statistical Problem Solving and Data Science Process

The statistical problem solving and data science process involves four major components statistically literate students consider and address as they investigate problems using statistics. Although the process for statistical problem solving is not sequential, all four components inform each other. Making statistically sound inferences and conclusions comes from building a strong investigative process for collecting and analyzing data.

The cycle below details the Statistical Problem Solving and Data Science Process. This Process was modified from the GAISE report. Further details on this Process from the GAISE Report be found at The Guidelines for Assessment and Instruction in Statistics Education (GAISE).



# Statistics and Probability Standards

## Domain: Interpreting Categorical and Quantitative Data (S.ID)

### Cluster: A. Summarize, represent and interpret data on a single count or measurement variable.

| Standard | Clarifications/Examples | Coherence |
|---|---|---|
| **S.ID.A.1**<br>Represent data with plots on the real number line (dot plots, histograms and box plots). | • Represent a data set using multiple plots. Identify key statistical information each plot provides.<br>• Informally discuss shape, center and spread of data using appropriate data displays.<br>• Emphasize which displays to use for discrete versus continuous data. | **Where students are coming from:**<br>• **6.SP.B.4:** Display numerical data in plots on a number line, including dot plots, histograms, and box plots.<br><br>**Where students are going:**<br>• **S.ID.A.2:** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.<br>• **S.ID.A.3:** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). |
| **S.ID.A.2**<br>Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. | • Focus on interpretation of data and decision-making based on analysis of data.<br>• **Note:** Standard deviation is the new learning in this standard. | **Where students are coming from:**<br>• **6.SP.B.5c:** Use measures of center (mean, median) and variability (IQR, MAD) to describe data.<br>• **7.SP.B.3:** Measure difference between centers of data sets by expressing it as a multiple of the measure of variability.<br><br>**Where students are going:**<br>• **S.ID.A.3:** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). |

| Standard | Clarifications/Examples | Coherence |
|---|---|---|
| **S.ID.A.3**<br>Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). | • Focus on interpretations in terms of real-world context.<br>• Emphasize which measures of center are resistant to extreme values.<br>• Differentiate between extreme values and outliers (i.e. how far out does an extreme value have to be in order to be considered an outlier?).<br>• Introduce the 1.5(IQR) rule for determining outliers. | **Where students are coming from:**<br>• **6.SP.A:** Develop an understanding of statistical variability.<br>• **6.SP.B:** Summarize and describe distributions.<br>• **7.SP.B.3:** Measure difference between centers of data sets by expressing it as a multiple of the measure of variability.<br><br>**Where students are going:**<br>• **S.ID.A.4:** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. |
| **S.ID.A.4**<br>Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. | • Focus on analysis and application, and not on tedious calculations.<br>• Introduce the Empirical Rule (68%-95%-99.7% Rule) and percentiles.<br>• Introduce *z*-scores as the number of standard deviations a data value is from the mean. Compare the extremity of data values from competing data sets using *z*-score values.<br>• Allow access to statistical software and technology, which are tools that students should use strategically. Include reading statistical summaries from various statistical software outputs.<br>• Recognize data sets for which procedures are not appropriate and normal distribution calculations and processes will not work (e.g., skewed distributions, non-normal, uniform, etc.). | **Where students are coming from:**<br>• **6.SP.B:** Summarize and describe distributions.<br>• **7.SP.A:** Use random sampling to draw inferences about a population.<br>• **7.SP.B:** Draw informal comparative inferences about two populations.<br>• **S.ID.A.2:** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.<br><br>**Where students are going:**<br>• **S.IC.A:** Understand and evaluate random processes underlying statistical experiments.<br>• **S.IC.B:** Make inferences and justify conclusions from sample surveys, experiments, and observational studies. |

## Cluster: B. Summarize, represent, and interpret data on two categorical and quantitative variables.

| Standard | Clarifications/Examples | Coherence |
|---|---|---|
| **S.ID.B.5**<br>Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. | • Refer to the wording of the standard. | **Where students are coming from:**<br>• **8.SP.A.4:** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.<br><br>**Where students are going:**<br>• **S.IC.A.1:** Understand statistics as a process for making inferences about population parameters based on a random sample from that population.<br>• **S.CP.A.4:** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. |
| **S.ID.B.6**<br>Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. | • Describe the trend and pattern (linear, quadratic or exponential) of data. | **Where students are coming from:**<br>• **8.SP.A.1:** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities.<br>• **HSS.ID.B.6:** (Algebra I and Algebra II) Summarize, represent, and interpret data on two categorical and quantitative variables.<br><br>**Where students are going:**<br>• **S.ID.B.6.a:** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. |

| Standard | Clarifications/Examples | Coherence |
|---|---|---|
| **S.ID.B.6.a**<br>Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. | • Standard S.ID.B.6.a focuses on using information such as a given function or the context of the problem to solve problems compared to standard S.ID.B.6.c that more specifically asks to fit a linear function to a scatter plot.<br>• Explore the uses and limitations of interpolation and extrapolation.<br>• Explore the connections between the population sampled and more generalized populations. | **Where students are coming from:**<br>• Fitting functions to data included linear, quadratic, and exponential models, as these are the function families studied in Algebra I and Algebra II.<br>• **8.SP.A.2:** Informally fit a straight line to data that suggests a linear association.<br><br>**Where students are going:**<br>• **S.ID.B.6.b:** Informally assess the fit of a function by plotting and analyzing residuals. |
| **S.ID.B.6.b**<br>Informally assess the fit of a function by plotting and analyzing residuals. | • Formally assess the fit of a function, including analyzing residual plots, correlation coefficients, and coefficient of determination by utilizing appropriate technology and statistical programs.<br>• Recognize and explore linear, quadratic and exponential patterns in data and notice such patterns in residual plots.<br>• Recognize that a relatively high correlation coefficient is no guarantee that the model is appropriate. | **Where students are coming from:**<br>• **S.ID.B.6.a:** Fit a function to the data. Use functions fitted to data to solve problems in the context of the data.<br><br>**Where students are going:**<br>• **S.ID.C.7:** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. |
| **S.ID.B.6.c**<br>Fit a linear function for a scatter plot that suggests a linear association. | • Recognize a linear relationship displayed in a scatter plot.<br>• Determine an equation for the line of best fit for a set of data points. | **Where students are coming from:**<br>• **S.ID.B.6.a:** Fit a function to the data. Use functions fitted to data to solve problems in the context of the data.<br>• **S.ID.B.6.b:** Informally assess the fit of a function by plotting and analyzing residuals.<br><br>**Where students are going:**<br>• **S.ID.C.7:** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.<br>• **S.ID.C.8:** Compute (using technology) and interpret the correlation coefficient of a linear fit. |

## Cluster: C. Interpret linear models.

| Standard | Clarifications/Examples | Coherence |
|---|---|---|
| **S.ID.C.7**<br>Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | • Analyze the reasonableness of the *y*-intercept in the context of the situation.<br>• Interpret slope in the context of the situation. | **Where students are coming from:**<br>• **8.SP.A.3:** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.<br>• **S.ID.B.6.c:** Fit a linear function for a scatter plot that suggests a linear association.<br>**Where students are going:**<br>• **S.ID.C.8:** Compute (using technology) and interpret the correlation coefficient of a linear fit. |
| **S.ID.C.8**<br>Compute (using technology) and interpret the correlation coefficient of a linear fit. | • Identify and interpret the correlation coefficient as a measure of how well the data fits the relationship.<br>• Comment on the strength and direction of the association in context of the variables.<br>• Include analyzing residual plots, correlation coefficients, and coefficient of determination. | **Where students are coming from:**<br>• **8.SP.A.2:** Know that straight lines are widely used to model relationships between two quantitative variables.<br>• **S.ID.B.6.b:** Informally assess the fit of a function by plotting and analyzing residuals.<br>**Where students are going:**<br>• **S.ID.9:** Distinguish between a statistical relationship and a cause-and-effect relationship.<br>• **S.IC.B:** Make inferences and justify conclusions from sample surveys, experiments, and observational studies. |
| **S.ID.C.9**<br>Distinguish between correlation and causation. | • Recognize a strong correlation does not indicate causation. | **Where students are coming from:**<br>• **S.ID.B.5:** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data. Recognize possible associations/trends in the data.<br>• **S.ID.B.6:** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.<br>**Where students are going:**<br>• **S.IC.B.3:** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. |

## Domain: Making Inferences and Justifying Conclusions (S.IC)

### Cluster: A. Understand and evaluate random processes underlying statistical experiments.

| Standard | Clarifications/Examples | Coherence |
|---|---|---|
| **S.IC.A.1**<br>Understand statistics as a process for making inferences about population parameters based on a random sample from that population. | • Emphasize the importance of randomization in selecting samples from a population, understanding that randomization guarantees that a process is unbiased.<br>• Understand the goal of randomization is to obtain a sample that is representative of the population.<br>• Understand the use of a sample statistic to measure a population parameter. | **Where students are coming from:**<br>• **7.SP.A.1:** Understand that statistics can be used to gain information about a population by examining a representative sample of the population.<br>• **7.SP.A.2:** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest.<br><br>**Where students are going:**<br>• **S.IC.B.3:** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.<br>• **S.IC.B.4:** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. |
| **S.IC.A.1.a**<br>Introduce sampling distributions as a means to explore variability in sample statistics and ultimately evaluate a claim about a population. | • Distinguish between a parameter and a statistic.<br>• Distinguish among the distribution of a population, the distribution of a sample, and the sampling distribution of a statistic.<br>• Use the sampling distribution of a statistic to evaluate a claim about a parameter.<br>• Describe the relationship between sample size and the variability of a statistic.<br>• Determine whether or not a statistic is an unbiased estimator of a population parameter. | **Where students are coming from:**<br>• **7.SP.A.1:** Understand that statistics can be used to gain information about a population by examining a representative sample of the population.<br>• **7.SP.A.2:** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest.<br><br>**Where students are going:**<br>• **S.IC.B.4:** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. |

| Standard | Clarifications/Examples | Coherence |
|---|---|---|
| **S.IC.A.2**<br>Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model*? | • Allow time for students to concretely simulate meaningful events in real life before moving to computer simulations.<br>• Use technology to help make connections between experimental and theoretical probability (i.e. the law of large numbers says that experimental probability will approach theoretical probability over a sufficiently large number of trials).<br>• Emphasize that the variability around the expected value is diminished when based on larger sample sizes.<br>• Establish connections to the ideas of independence of events and simulations. | **Where students are coming from:**<br>• **7.SP.C.7:** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.<br>• **7.SP.C.8.c:** Design and use a simulation to generate frequencies for compound events.<br><br>**Where students are going:**<br>• **S.IC.B.4:** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.<br>• **S.IC.B.5:** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. |

## Cluster: B. Make inferences and justify conclusions from sample surveys, experiments and observational studies.

| Standard | Clarifications/Examples | Coherence |
|---|---|---|
| **S.IC.B.3**<br>Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. | • Develop procedures for random selection and assignment that extend further than a classroom census either with simulation or within larger populations to make appropriate inferences.<br>• Determine the impact of how questions are asked and how accurately responses are recorded on sample surveys.<br>• Identify types of bias and emphasize their impact on the conclusion made.<br>• Recognize how aspects in the design of an experiment or observational study influence the conclusions that can be drawn.<br>• Develop understanding of sampling techniques including but not limited to convenience sampling, voluntary response sampling, simple random sampling, stratified random sampling, and cluster sampling. | **Where students are coming from:**<br>• **7.SP.A:** Use random sampling to draw inferences about a population.<br>• **S.IC.A.1**: Understand statistics as a process for making inferences about population parameters based on a random sample from that population.<br><br>**Where students are going:**<br>• **S.IC.B.4:** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.<br>• **S.IC.B.5:** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. |
| **S.IC.B.4**<br>Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. | • Create sampling distributions through simulation with technology or in-class experiments.<br>• Use technology to provide opportunities for students to explore multiple situations.<br>• Emphasize the factors that influence the margin of error, such as sample size and confidence levels.<br>• Establish a clear distinction between estimates for means and proportions.<br>• Explore confidence levels and confidence intervals.<br>• Explore random sampling with replacement, also known as bootstrapping, to develop the margin of error. | **Where students are coming from:**<br>• **S.IC.A.1**: Understand statistics as a process for making inferences about population parameters based on a random sample from that population.<br>• **S.IC.A.2:** Decide if a specified model is consistent with results from a given data-generating process.<br>• **S.IC.B.3:** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.<br><br>**Where students are going:**<br>• **S.IC.B.6:** Evaluate reports based on data. |

<table>
<tr><th>Standard</th><th>Clarifications/Examples</th><th>Coherence</th></tr>
<tr><td>S.IC.B.5<br>Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.</td><td>• Use data from well-designed experiments (treatments, control groups, experimental units, random assignments and replications) to conduct informal hypothesis tests.<br>• Compare two or more samples of data to infer whether there is a real difference between the populations from which they were sampled.<br>• Make generalizations that go beyond describing the given data, using given data as evidence.<br>• Form conclusions, recognizing and expressing degrees of uncertainty.<br>• Judge two competing models or statements to informally determine which is more likely to be true.</td><td>Where students are coming from:<br>• 7.SP.B.4: Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.<br>• S.IC.A.1: Understand statistics as a process for making inferences about population parameters based on a random sample from that population.<br>• S.IC.A.2: Decide if a specified model is consistent with results from a given data-generating process.<br>• S.IC.B.3: Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.<br><br>Where students are going:<br>• S.IC.B.6: Evaluate reports based on data.</td></tr>
<tr><td>S.IC.B.6<br>Evaluate reports based on data.</td><td>• Use and evaluate statistical language appropriately. For example, avoid using absolute language, such as "will happen."<br>• Evaluate graphs, tables, and reports to identify possibly misleading characteristics.<br>• Evaluate how randomness is applied in real world articles and studies.<br>• Interpret and critically evaluate statistical information and data-based arguments from a diverse range of sources.</td><td>Where students are coming from:<br>• 6.SP.B.5: Summarize numerical data sets in relation to their context.<br>• S.IC.B.4: Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.<br>• S.IC.B.5: Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.<br><br>Where students are going:<br>• IHE: Formally conduct hypothesis tests and formally establish confidence intervals.</td></tr>
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| Standard | Clarifications/Examples | Coherence |
| --- | --- | --- |
| **S.IC.B.7**<br>Conduct statistical investigations. | • Apply the Statistical Problem Solving Process (i.e. formulate statistical questions, collect and consider the data, analyze the data, and interpret the results).<br>• Distinguish between random assignment and random selection and their implications on the conclusions that can be drawn. | **Where students are coming from:**<br>• **7.SP.A:** Use random sampling to draw inferences about a population.<br>• **S.IC.A.1:** Understand statistics as a process for making inferences about population parameters based on a random sample from that population.<br><br>**Where students are going:**<br>• **S.IC.B.4:** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.<br>• **S.IC.B.5:** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. |
| **S.IC.B.7.a**<br>Conduct observational studies. | • Plan and conduct well-designed observational studies by applying methods of random selection and limiting bias when possible. | **Where students are coming from:**<br>• **7.SP.A:** Use random sampling to draw inferences about a population.<br>• **S.IC.A.1:** Understand statistics as a process for making inferences about population parameters based on a random sample from that population.<br><br>**Where students are going:**<br>• **S.IC.B.4:** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.<br>• **S.IC.B.5:** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. |

| Standard | Clarifications/Examples | Coherence |
|---|---|---|
| **S.IC.B.7.b**<br>Conduct statistical experiments. | • Plan and conduct well-designed experiments (i.e. random assignment to treatment and control group, etc.). | **Where students are coming from:**<br>• **7.SP.A:** Use random sampling to draw inferences about a population.<br>• **S.IC.A.1:** Understand statistics as a process for making inferences about population parameters based on a random sample from that population.<br><br>**Where students are going:**<br>• **S.IC.B.4:** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.<br>• **S.IC.B.5:** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. |

## Domain: Conditional Probability and the Rules of Probability (S.CP)

### Cluster: A. Understand independence and conditional probability and use them to interpret data.

| Standard | Clarifications/Examples | Coherence |
|---|---|---|
| **S.CP.A.1**<br>Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or", "and", "not"). | • Explore sample spaces and probability models using various displays (i.e. tree diagrams, Venn diagrams and two-way tables).<br>• Represent events using appropriate notation (e.g., $\cap$, $\cup$, ~ , $^c$, $\in$). | **Where students are coming from:**<br>• **7.SP.C.8:** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulations.<br><br>**Where students are going:**<br>• **S.CP.A.2:** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.<br>• **S.CP.A.3:** Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. |
| **S.CP.A.2**<br>Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. | • Build understanding through real world contexts and experiments before advancing to theoretical and formulaic approaches (*e.g., experiment with flipping a coin or playing egg roulette*).<br>• Develop an understanding of the formula for determining if events are independent. | **Where students are coming from:**<br>• **7.SP.C.7:** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.<br>• **7.SP.C.8:** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.<br><br>**Where students are going:**<br>• **S.CP.A.3:** Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.<br>• **S.CP.B.8:** Apply the general Multiplication Rule in a uniform probability model, and interpret the answer in terms of the model. |

| Standard | Clarifications/Examples | Coherence |
| --- | --- | --- |
| **S.CP.A.3**<br>Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. | • This standard should be taught in conjunction with S.CP.A.4 and S.CP.A.5 to build coherence and understanding of conditional and independent probabilities.<br>• Attention should be paid to the coherence of the standards for the cluster S.CP.A.<br>• Use the formula, Venn Diagrams, and properties for conditional probability to determine if two events are independent or dependent. | **Where students are coming from:**<br>• **7.SP.C:** Investigate chance processes and develop, use, and evaluate probability models.<br>• **S.CP.A:** Understand independence and conditional probability and use them to interpret data.<br><br>**Where students are going:**<br>• **S.CP.A.1:** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or", "and", "not").<br>• **S.CP.A.2:** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.<br>• **S.CP.B.8:** Apply the general Multiplication Rule in a uniform probability model, and interpret the answer in terms of the model. |
| **S.CP.A.4**<br>Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* | • This standard should be taught in conjunction with S.CP.A.3 and S.CP.A.5 to build coherence and understanding of conditional and independent probabilities.<br>• Attention should be paid to the coherence of the standards for the cluster S.CP.A.<br>• Analyze a table to calculate and interpret marginal and conditional probabilities and any association or independence that may exist. | **Where students are coming from:**<br>• **7.SP.C:** Investigate chance processes and develop, use, and evaluate probability models.<br>• **8.SP.A.4:** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.<br>• **S.CP.A:** Understand independence and conditional probability and use them to interpret data.<br><br>**Where students are going:**<br>• **S.CP.A.1:** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or", "and", "not").<br>• **S.CP.A.2:** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. |

| Standard | Clarifications/Examples | Coherence |
|---|---|---|
| **S.CP.A.5**<br>Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* | • This standard should be taught in conjunction with S.CP.A.3 and S.CP.A.4 to build coherence and understanding of conditional and independent probabilities.<br>• Make connections between the wording in the scenario and the formula $P(A\|B) \neq P(B\|A)$. | **Where students are coming from:**<br>• **7.SP.C:** Investigate chance processes and develop, use, and evaluate probability models.<br>• **S.CP.A:** Understand independence and conditional probability and use them to interpret data.<br><br>**Where students are going:**<br>• **S.CP.A.1:** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or", "and", "not").<br>• **S.CP.A.2:** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. |

## Cluster: B. Use the rules of probability to compute probabilities of compound events.

| Standard | Clarifications/Examples | Coherence |
|---|---|---|
| **S.CP.B.6**<br>Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. | • Connect addition and multiplication rules with counting outcomes.<br>• Present data in multiple representations (i.e. tables, tree diagrams, Venn Diagrams and lists). | **Where students are coming from:**<br>• **S.CP.A:** Understand independence and conditional probability and use them to interpret data.<br><br>**Where students are going:**<br>• **S.CP.B.7:** Apply the Addition Rule, and interpret the answer in terms of the model.<br>• **S.CP.B.8:** Apply the Multiplication Rule in a uniform probability model, and interpret the answer in terms of the model. |
| **S.CP.B.7**<br>Apply the Addition Rule,<br>$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. | • Connect addition and multiplication rules with counting outcomes.<br>• Present data in multiple representations (i.e. two-way tables, Venn Diagrams and lists).<br>• Develop conceptual understanding of the addition rule before introducing the formula.<br>• Distinguish between mutually exclusive events, disjoint events, and independent events.<br>• Show how the addition rule can be applied when events are mutually exclusive. | **Where students are coming from:**<br>• **S.CP.B.6:** Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.<br><br>**Where students are going:**<br>• **S.CP.B.8:** Apply the Multiplication Rule in a uniform probability model, and interpret the answer in terms of the model. |

| Standard | Clarifications/Examples | Coherence |
|---|---|---|
| **S.CP.B.8**<br>Apply the general Multiplication Rule in a uniform probability model,<br>*P(A and B) = P(A)P(B\|A) = P(B)P(A\|B)*, and interpret the answer in terms of the model. | • Connect addition and multiplication rules with counting outcomes.<br>• Present data in multiple representations (i.e. tables, Venn Diagrams and lists).<br>• Explore false positives and false negatives that arise from multiple contexts using conditional probability.<br>• Show how the multiplication rule can be applied when events are independent. | **Where students are coming from:**<br>• **S.CP.A.3:** Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.<br>• **S.CP.A.5:** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.<br>• **S.CP.B.6:** Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.<br>• **S.CP.B.7:** Apply the Addition Rule, and interpret the answer in terms of the model.<br><br>**Where students are going:**<br>• **S.MD:** Apply probability models and formulas to make decisions. |
| **S.CP.B.9**<br>Use permutations and combinations to compute probabilities of compound events and solve problems. | • Perform simple calculations by hand to develop understanding of permutations and combinations.<br>• Use technology to perform calculations when computing permutations and combinations to solve more complex problems.<br>• Connect to the fundamental counting principle. | **Where students are coming from:**<br>• **S.CP.A:** Understand independence and conditional probability and use them to interpret data.<br><br>**Where students are going:**<br>• **S.MD:** Apply probability models and formulas to make decisions. |

## Domain: Using Probability to Make Decisions (S.MD)

### Cluster: A. Calculate expected values and use them to solve problems.

| Standard | Clarifications/Examples | Coherence |
|---|---|---|
| **S.MD.A.1**<br>Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. | • Construct probability distributions as dot plots, stacked bar graphs, frequency line graphs, mosaic plots and box plots for categorical data.<br>• Use simulations and real world data to build sample spaces.<br>• Distinguish between discrete and continuous random variables. | **Where students are coming from:**<br>• **S.IC.B.6:** Evaluate reports based on data.<br>• **S.ID.A.1:** Represent data with plots on the real number line (dot plots, histograms and box plots).<br><br>**Where students are going:**<br>• **S.IC.A:** Understand and evaluate random processes underlying statistical experiments.<br>• **S.IC.B:** Make inferences and justify conclusions from sample surveys, experiments, and observational studies. |
| **S.MD.A.2**<br>Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. | • Relate expected value to distributions of samples and confidence intervals.<br>• Use experiments, patterns, and theoretical probability distributions to develop and interpret the formula for expected value.<br>• Apply the expected value to real world situations that require finding the mean of the distribution.<br>• Distinguish between binomial and geometric random variables. | **Where students are coming from:**<br>• **S.ID.A.2:** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.<br>• **S.MD.A.1:** Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.<br><br>**Where students are going:**<br>• **S.MD.A.3:** Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.<br>• **S.MD.B.5.b:** Evaluate and compare strategies on the basis of expected values. |

| Standard | Clarifications/Examples | Coherence |
|---|---|---|
| **S.MD.A.3**<br>Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* | • Experiment with common real world probabilities (e.g. playing cards) to calculate and apply expected value.<br>• Explore different approaches to find probabilities of events (short term vs. long term). | **Where students are coming from:**<br>• **S.MD.A.1:** Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.<br><br>**Where students are going:**<br>• **S.MD.A.4:** Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. |
| **S.MD.A.4**<br>Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* | • Apply the Data Science Process to:<br>  ○ formulate informed questions.<br>  ○ collect, analyze, and report out on data.<br>  ○ make informed decisions based on probability models. | **Where students are coming from:**<br>• **S.MD.A.3:** Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.<br><br>**Where students are going:**<br>• **S.MD.B:** Use probability to evaluate outcomes of decisions. |

## Cluster: B. Use probability to evaluate outcomes of decisions.

| Standard | Clarifications/Examples | Coherence |
|---|---|---|
| **S.MD.B.5**<br>Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. | See S.MD.B.5.a and S.MD.B.5.b. | **Where students are coming from:**<br>• **7.SP.C**: Investigate chance processes and develop, use, and evaluate probability models.<br>**Where students are going:**<br>• **S.MD.B:** Use probability to evaluate outcomes of decisions. |
| **S.MD.B.5.a**<br>Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* | • Analyze the benefits or drawbacks when weighing a decision based on money or another valuable item that can be numerically quantified.<br>• Interpret expected value as the long-run expected outcome of the variable, taking into account the probability of each possible outcome. | **Where students are coming from:**<br>• **7.SP.C:** Investigate chance processes and develop, use, and evaluate probability models.<br>**Where students are going:**<br>• **S.MD.B:** Use probability to evaluate outcomes of decisions. |
| **S.MD.B.5.b**<br>Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.* | • Interpretations of expected value should be thought of as what is expected in the long run.<br>• Explore the notion of 'Monte Carlo Fallacy' (the mistaken belief that if something happens less frequently than normal, it will happen more often in the future).<br>• Work with various simulation tools such as dice, coins, roulette wheels, spinners and other simulation tools to assign particular payoffs and costs for play. | **Where students are coming from:**<br>• **7.SP.C:** Investigate chance processes and develop, use, and evaluate probability models.<br>**Where students are going:**<br>• **S.MD.B:** Use probability to evaluate outcomes of decisions. |
| **S.MD.B.6**<br>Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). | • Explore the use of different tools to generate random samples and explore the fairness of random selection processes.<br>• Students should be required to simulate events. Focus student attention towards the context of the situation and how this is represented with the chosen tool or simulation. | **Where students are coming from:**<br>• **S.MD.B.5:** Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. |
| **S.MD.B.7**<br>Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). | • Analyze both cost and benefits in different contextual situations.<br>• Explore negative values occurring when the cost outweighs the benefit in certain situations. | **Where students are coming from:**<br>• **S.MD.B.5:** Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. |

## Resources

The following documents and resources have been included for reference when developing courses.

- GAISE II Report:
    - This report from the American Statistical Association is an extension of the GAISE I Report that was written in conjunction with NCTM to enhance the statistical standards.
- Achieve the Core Coherence Maps
    - The coherence maps from Achieve the Core highlight the connections between the statistics standards at the high school level and the standards from prior grade levels.
- Mathematics Foundations for Success in Introductory Statistics
    - This document from the DANA Center highlights how common mathematical skills lay a foundation for a statistics and probability course.
- First in the World Maryland Mathematics Reform Initiative (FITW MMRI)
    - The work associated with the FITW MMRI resulted in the development of a mathematics pathway offered by Maryland's Institutions of Higher Education (IHE) that includes Statistics as the first credit-bearing college-level mathematics course available to students attending Maryland's IHEs.
    - The work associated with the FITW MMRI included the development of a non-credit course entitled Mathematical Foundations for students determined as not possessing the prerequisite mathematics skills needed for success in the entry level Statistics course.
    - Additional resources to use for the development of a Statistics and Probability course include the Mathematics Foundations for Success in Introductory Statistics from the Dana Center.